A direct proof of implication's transitivity comes from working with the full proposition simultaneous, and showing it reduces to True.

Proof. □

$((p \to q) \wedge (q \to r)) \to (p \to r)$	given
$\neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r)$	definition of implication
$(\neg(\neg p \vee q) \vee \neg(\neg q \vee r)) \vee (\neg p \vee r)$	DeMorgan's Laws
$((\neg\neg p \wedge \neg q) \vee (\neg\neg q \wedge \neg r)) \vee (\neg p \vee r)$	DeMorgan's Laws
$((p \wedge \neg q) \vee (q \wedge \neg r)) \vee (\neg p \vee r)$	Double negative
$((p \wedge \neg q) \vee \neg p) \vee ((q \wedge \neg r) \vee r)$	Associativity and Commutativity
$((p \vee \neg p) \wedge (\neg q \vee \neg p)) \vee ((q \vee r) \wedge (\neg r \vee r))$	Distributive
$(T \wedge (\neg q \vee \neg p)) \vee ((q \vee r) \wedge T)$	Excluded Middle
$(\neg q \vee \neg p) \vee (q \vee r)$	Absorption
$\neg q \vee q \vee \neg p \vee r$	Associativity and Commutativity
$T \vee \neg p \vee r$	Excluded Middle
T	Absorption