Division theorem details

The division theorem says the following:

> For every positive integers a and every integer b, there is a unique pair of integers (q, r) where $b = aq + r$ and $0 \leq r < a$.

The division theorem has a number of details involved in it that are important, though they may not seem so until you look closely. For each of the following, determine if the statement is true or false. If it is false, provide a counterexample. If it is true, explain.

- For every pair of positive integers a and b, there is a unique quotient q and remainder r where $b = aq + r$ and $0 \leq r < a$.
- For every pair of positive integers a and b, there is a unique pair of integers (q, r) where $b = aq + r$.
- For every pair of positive integers a and b, there is a unique pair of integers (q, r) where $b = aq + r$ and $0 < r < a$.
- For every pair of positive integers a and b, there is a unique pair of integers (q, r) where $b = aq + r$ and $0 \leq r \leq a$.
- For every pair of positive integers a and b, there is a unique pair of integers (q, r) where $b = aq + r$ and $0 \leq r < a$.
- For every number b and positive integer a, there is a unique pair of integers q and r where $b = aq + r$ and $0 \leq r < a$.
- For every pair of integers (a, b), there is a unique pair of integers (q, r) where $b = aq + r$ and $0 \leq r < a$.
- For every pair of integers (a, b) with $a \neq 0$, there is a unique pair of integers (q, r) where $b = aq + r$ and $0 \leq r < |a|$.