Homework 11, Math 251-010

Thursday, April 19, 2012, due April 25, 2012

NOTE: There was a typo in problem 3. This is now corrected.

This homework covers material from Sections 10.5-10.7. You should practice the problems at the ends of these sections before starting this homework if you really want to understand things.

1. Suppose $u_t = 9u_{xx}$ in $[0, 2]$ with homogeneous Neumann boundary conditions and
$$u(x, 0) = 2 - \cos(\pi x/2) + \cos(3\pi x)/5.$$
Find $u(x, t)$.

2. True or False: Suppose $u(x, t)$ solves a heat equation with Robin boundary conditions on the interval $[0, L]$. Then
$$\lim_{t\to\infty} u'(0, t) = \lim_{t\to\infty} u'(L, t).$$
Explain.

3. Suppose $u_t = 9u_{xx}$ in $[0, 2]$ with generalized Robin boundary conditions
$$u(0, t) + u_x(0, t) = 4,$$
$$u(2, t) - 2u_x(2, t) = 1.$$
Find $\lim_{t\to\infty} u(x, t)$.

4. Suppose $u(x, t)$ is a solution of the wave equation $u_{tt} = 4u_{xx}$ on the interval $[0, 3]$ with ends fixed at $u(0, t) = u(3, t) = 0$, and that initially, $u(x, 0) = \min(x, 3 - x)$ with no initial velocity. Find the complete solution $u(x, t)$ in terms of an infinite Fourier series.

5. Suppose $u(x, t)$ is a solution of the wave equation $u_{tt} = 5u_{xx}$ on the interval $[0, 3]$ with ends fixed at $u(0, t) = u(3, t) = 0$, and that initially $u_t(x, 0) = \sin(\pi x/3)$ with no initial displacement. Find the complete solution $u(x, t)$ in terms of a Fourier series.

6. Problem 16, section 10.7.