Homework 11, Math 251-010

Wednesday, April 11, 2012, due April 18, 2012

This homework covers material from Sections 10.1-10.5. You should practice the problems at the ends of these sections before starting this homework if you really want to understand things.

1. Linear autonomous 2nd-order boundary-value problems
   (a) If $y'' - 4y = 0$, $y(-1) = 1$, $y(1) = -1$, find all solutions $y(x)$.
   (b) If $y'' + 2y' + 5y = 0$, $y(0) = 3$, $y(\pi/2) = -1$, find all solutions $y(x)$.
   (c) If $y'' + 2y' + 5y = 0$, $y(0) = 3$, $y(\pi/2) = -3e^{-\pi/2}$, find all solutions $y(x)$.
   (d) Find all the eigenvalues λ for which $y'' + \lambda y = 0$ with $y(0) = 0$ and $y'(\pi) = 0$ admits solutions on the interval $[0, \pi]$.
2. Fourier series -- Draw each period function and calculate Fourier-series representations.
   (a) $f(x) = f(x + 6)$ and
   $$f(x) = \begin{cases} 1 & \text{if } -1 < x \leq 1 \\ 0 & \text{if } 1 < x^2 < 9 \end{cases}$$
   (b) $f(x) = f(x + 6)$ and
   $$f(x) = \begin{cases} x & \text{if } -3 < x \leq 2 \\ 0 & \text{if } 2 \leq x < 3 \end{cases}$$
3. Let $h(x) = \max(0, 2 - x)$ when $0 \leq x \leq 4$. Plot an odd 2L-periodic extensions and an even 2L-periodic extensions. Calculate a sine-series representation of the odd extension and a cosine-series representation of the even extension.
4. The heat equation.
   (a) Is $\cos(2x)e^{-2t}$ a solution of the equation
   $$\dot{u} = \frac{1}{2}u''$$
   with boundary conditions $u'(0, t) = 0$, $u'(\pi, t) = 0$? Show your work.
5. Use separation of variables to transform the partial differential equation (PDE)
   $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + k\frac{\partial u}{\partial x}$$
   into a system of two ordinary differential equations if you can.
6. Use separation of variables to transform the PDE
   $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + k\frac{\partial^2 u}{\partial x \partial t}$$
   into a system of two ordinary differential equations if you can.
7. Use separation of variables to construct a Fourier-series representation for a function $u(x, t)$ solving the cable equation $\dot{u} = Du'' - ru$, with boundary conditions $u(K, t) = u(-K, t) = 0$ and initial condition $u(x, 0) = f(x)$.