Homework 10, Math 251-010

Wednesday, March 27, 2012, due April 4, 2012

This home covers material from Sections 9.1, 9.2 and 9.3. You should practice the problems at the ends of these sections before starting this homework if you really want to understand things.

1. A special case of the Van der Pol equation can be written in system-form as

$$\begin{aligned} \dot{x} &= 3x - x^3 - 3y \\ \dot{y} &= x/3. \end{aligned}$$

   Show that the point $(0,0)$ is a stationary solution, and classify the behavior of solutions near this point.

2. Lotka's model of competition between two species is given by the system

$$\begin{aligned} \dot{x} &= px(1 - ax - by), \\ \dot{y} &= ry(1 - cx - dy). \end{aligned}$$

   If $p = 2$, $r = 3$, $a = 1/3$, $b = 1/4$, $c = 1/4$, $d = 1/3$, find four stationary solutions. Classify each of these stationary solutions, and determine their stability.

3. The Duffing equation represents the motion of a mass under a restoring force that behaves like a linear Hooke's Law for small displacements, but is curved for larger displacements. It is

$$\ddot{x} + \gamma\dot{x} + k(1 + ax^2)x = 0.$$

   (a) Rewrite Duffing's equation as a system of two first-oder differential equations.

   (b) If $\gamma = 1/2$, $k = 4$ and $a = -1/9$, find 3 stationary solutions to this system.

   (c) Use eigenvalues to classify these 3 stationary solutions. Then draw a cartoon phase-plane of the system, using eigenvectors where appropriate.

4. Study for the second exam. Old exams are at `http://www.math.psu.edu/tseng/class/M251samples.html` A study guide is at `http://www.math.psu.edu/treluga/251/MATH251_SP2012_exam_2_guide.pdf`