Homework 9, Math 251-010

Wednesday, March 21, 2012, due March 28, 2012

This home covers material from Sections 7.5, 7.6, 7.8, and 9.1. You *must* practice the problems at the ends of these sections before starting this homework if you really want to understand things.

1. Consider each of the following first order linear systems
   - Find the general solution of this system.
   - Find the specific solution if $y(0) = [2, 1]$.

   (a)
   $$\frac{dy}{dt} = Ay, \quad \text{where} \quad A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}.$$

   (b)
   $$\frac{dy}{dt} = Ay, \quad \text{where} \quad A = \begin{bmatrix} 3 & -1 \\ -24 & -2 \end{bmatrix}.$$

   (c)
   $$\frac{dy}{dt} = Ay, \quad \text{where} \quad A = \begin{bmatrix} 13 & 8 \\ -18 & -12 \end{bmatrix}.$$

   (d)
   $$\frac{dy}{dt} = Ay, \quad \text{where} \quad A = \begin{bmatrix} -6 & -1 \\ 4 & -6 \end{bmatrix}.$$

   (e)
   $$\frac{dy}{dt} = Ay, \quad \text{where} \quad A = \begin{bmatrix} -1 & -6 \\ 3 & 5 \end{bmatrix}.$$

   (f)
   $$\frac{dy}{dt} = Ay, \quad \text{where} \quad A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}.$$

   (g)
   $$\frac{dy}{dt} = Ay, \quad \text{where} \quad A = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix}.$$

   (h)
   $$\frac{dy}{dt} = Ay, \quad \text{where} \quad A = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}.$$

2. For each of the problems above, draw a phase-plane representation of the dynamics around that stationary solution.

3. Classify the type and stability of the stationary solution $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as a saddle point, a stable node, an unstable node, a stable focus, an unstable focus, or a center.

4. What's the difference between a solution being stable and a solution being asymptotically stable?