

Homework 8, Math 251-010

Wednesday, March 14, 2012, due March 21, 2012

This home covers material from Sections 6.5, 7.1, 7.2, and 7.3. You *should* practice the problems at the ends of these sections before starting this homework.

1. Find the specific solution to the following initial-value problems using Laplace--Heaviside methods.

(a)
$$y'' + 4y' + 3y = \delta(x - 12), \quad y(0) = 0, \quad y'(0) = 0.$$

(b)
$$\ddot{y} + y = \delta(t + 2) + u_3(t), \quad y(0) = 0, \quad \dot{y}(0) = 0.$$

(c)
$$\ddot{y} + 20\dot{y} + 109y = \delta(t - 1), \quad y(0) = 2, \quad \dot{y}(0) = 0.$$

2. Transform the following equations into systems of 1st order ODE's.

(a)
$$\ddot{y} - 5\dot{y} + y = 0.$$

(b)
$$\ddot{y} - y\dot{y} + y^3 = t.$$

(c)
$$t^2 \ddot{y} - \dot{y} + ty = 3, \quad y(0) = \dot{y}(0) = 1.$$

3. What do theorems 7.1.1 and 7.1.2 say?

4. Section 7.2, 371-373, # 1, 22, 24

5. Give an example of a 4x4 symmetric matrix that is not diagonal. What do we know about the eigenvalues of this matrix?

6. Find the eigenvalues and eigenvectors of the following matrices.

(a)
$$\begin{bmatrix} -16 & -54 \\ 4 & 14 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 70 & -60 \\ 72 & -62 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 3 & \frac{1}{2} & 5 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 2 & 5 \\ -5 & 2 \end{bmatrix}$$

Challenge:

1. What's the difference between the solutions of

$$\ddot{y} + b\dot{y} + cy = \delta(t), \quad y(0) = 0, \quad \dot{y}(0) = 0 \quad \text{and} \quad \ddot{u} + b\dot{u} + cu = 0, \quad u(0) = 0, \quad \dot{u}(0) = 1?$$