Homework 7, Math 251-010

Monday, February 27, 2012, due March 14, 2012

This home covers material from Sections 6.1, 6.2, 6.3, and 6.4. You should practice the problems at the ends of these sections before starting this homework.

1. Indefinite integrals. Classify the behavior of the following indefinite integrals. For integrals that do not converge, determine if the integral diverges to ∞, $-\infty$, or if there is a set of values among which the integral is indeterminate.

   (a)
   $$\int_0^\infty xe^{-x}dx$$

   (b)
   $$\int_0^\infty -x^{10}e^{-x}dx$$

   (c)
   $$\int_0^\infty \cos(x+3)dx$$

   (d)
   $$\int_0^\infty x\sin(x^2)dx$$

   (e)
   $$\int_0^\infty \frac{2+x}{3+x^2}dx$$

   (f)
   $$\int_0^\infty \sin(3x)e^x dx$$

2. Calculate the following Laplace transforms, and provide a range of values for s for which the transform exists. Use tables and rules where-ever possible to simplify your work, but remember to cite your methods.

   (a) $\mathcal{L}\{4e^{13t}\}$
   (b) $\mathcal{L}\{\sinh(5t)\}$
   (c) $\mathcal{L}\{-15t^4\}$
   (d) $\mathcal{L}\{e^{6t}\sin(8t)\}$
   (e) $\mathcal{L}\{te^t + t\cos(t)\}$

3. Show that
   $$\frac{\partial}{\partial s}\mathcal{L}\{f(t)\} = -\mathcal{L}\{tf(t)\}.$$
   Use this theorem to calculate $\mathcal{L}\{t\sin(4t)\}$.

4. Inverting Laplace transforms requires practice. Calculate the following inverses.

   (a)
   $$\mathcal{L}^{-1}\left\{\frac{4}{s^3} - \frac{18}{s+3} + 2\right\}$$

(b)

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2-4s-21}\right\}$$

(c) Use completing-the square to calculate the following inverse.

$$\mathcal{L}^{-1}\left\{\frac{-6}{s^2-8s+25}\right\}.$$

(d)

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3-1}\right\}$$

5. Solve the following initial-value problems using Laplace-transform methods.

(a)

$$\ddot{c}-4\dot{c}-21c=0, \quad c(0)=4, \quad \dot{c}(0)=8.$$

(b)

$$\ddot{c}-4\dot{c}-21c=t, \quad c(0)=4, \quad \dot{c}(0)=8.$$

(c)

$$\ddot{c}-4\dot{c}-21c=e^{-t}, \quad c(0)=4, \quad \dot{c}(0)=8.$$

(d)

$$\ddot{y}-2\dot{y}+29y=e^{-t}, \quad y(0)=1, \quad \dot{y}(0)=0.$$

(e)

$$\dddot{x}-x=0, \quad \ddot{x}(0)=1, \quad x(0)=\dot{x}(0)=\dddot{x}(0)=0.$$

6. Represent the following functions in terms of polynomials and Heaviside functions.

(a)

$$f(x)=\begin{cases}0 & \text{if } 0\le x<1\\ 1 & \text{if } 1\le x<2\\ -1 & \text{if } 2\le x<3\\ -4 & \text{if } 3\le x\end{cases}$$

(b) $g(x)=|x-2|+|x+2|$

7. Solve the following initial-value problems using Laplace transforms.

(a)

$$\dot{y}-3y=u_2(t)+u_{10}(t), y(0)=\dot{y}(0)=0$$

(b)

$$\ddot{y}+4y=u_2(t)-u_3(t), y(0)=\dot{y}(0)=0$$

(c)

$$\ddot{y}+3\dot{y}+2y=1-u_9(t), y(0)=\dot{y}(0)=0$$