Homework 5, Math 251-010

Monday, February 13, 2012, due February 22, 2012

This home covers material from Sections 3.3, 3.4, 3.5, and 3.6.

1. Consider the differential equation

$$49\,\mathrm{x}(t) + 14\frac{\partial}{\partial t}\,\mathrm{x}(t) + \frac{\partial^2}{\partial^2 t}\,\mathrm{x}(t) = 0.$$

   (a) Find the characteristic equation.
   (b) Find a fundamental solution set.
   (c) Calculate the Wronksian and show that it is non-zero.
   (d) Construct the general solution.

2. Consider the second-order homogeneous equation with variable coefficients

$$(2x+1)\,\frac{\partial^2}{\partial^2 x}\,\mathrm{y}(x) + \left(4x^2+12x+3\right)\frac{\partial}{\partial x}\,\mathrm{y}(x) + \left(8x^2+16x+2\right)\mathrm{y}(x) = 0.$$

   (a) Use direct substitution to find the value of r for which e^{rx} is a solution to this equation. (Note: The factorization may require persistence.)
   (b) Use reduction-of-order to construct the general solution of this equation.

3. Consider the differential equation

$$-48\,\mathrm{y}(x) - 8\frac{\partial}{\partial x}\,\mathrm{y}(x) + \frac{\partial^2}{\partial^2 x}\,\mathrm{y}(x) = 576e^{-6x}, \quad \mathrm{y}(0) = 27, \quad \mathrm{y'}(0) = -252.$$

   (a) Find the general solution of the homogenized equation.
   (b) Find a particular solution using the method of undetermined coefficients.
   (c) Find the specific solution to the given problem.

4. Consider the differential equation

$$-15\,\mathrm{y}(x) + 2\frac{\partial}{\partial x}\,\mathrm{y}(x) + \frac{\partial^2}{\partial^2 x}\,\mathrm{y}(x) = -160\sin(5x) + 40\cos(5x), \quad \mathrm{y}(0) = -9, \quad \mathrm{y'}(0) = 1.$$

   (a) Find the general solution of the homogenized equation.
   (b) Find a particular solution using the method of undetermined coefficients.
   (c) Find the specific solution to the given problem.

5. Consider the differential equation

$$36\,\mathrm{y}(x) - 9\frac{\partial}{\partial x}\,\mathrm{y}(x) - \frac{\partial^2}{\partial^2 x}\,\mathrm{y}(x) = 36x^2 - 126x + 241, \quad \mathrm{y}(0) = 2, \quad \mathrm{y'}(0) = 30.$$

   (a) Find the general solution of the homogenized equation.
   (b) Find a particular solution using the method of undetermined coefficients.
   (c) Find the specific solution to the given problem.

6. Use variation of parameters to find the general solution of

$$\ddot{x} + x = \tan t \quad \text{when } -\pi/2 < t < 0.$$

7. Use variation of parameters to find a solution of

$$(1-t)\ddot{x} + t\dot{x} - x = (t-1)^2 e^{-2t}.$$

   Hint: A fundamental solution set is $\{e^t, t\}$.