Homework 4, Math 251-010

Wednesday, February 1, 2012, due February 8, 2012

This home covers material from Sections 3.1, 3.2, and 3.3.

1. Find the general solution of $y'' - 2y' - 35y = 0$.
2. Find the asymptotic behavior of the specific solution of the initial-value problem $\ddot{x} + 2\dot{x} - 8x = 0$, $x(0) = 5$, $\dot{x}(0) = -8$.
3. Find a fundamental solution set of
$$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} = 0$$
4. Section 3.2, problem 5.
5. Section 3.2, problem 9.
6. Section 3.2, problem 25.
7. Section 3.2, problem 31.
8. Section 3.3, problem 1.
9. Section 3.3, problem 8.
10. Section 3.3, problem 19.
11. You should also start studying for the first exam. http://www.math.psu.edu/tseng/class/M251samples.html We'll split up the work-load.
    (a) Last names AA - Ca, read Fall, 2011
    (b) Last names Cb - F, read Summer, 2011
    (c) Last names G - K, read Spring, 2011
    (d) Last names L - M, read Fall, 2010
    (e) Last names N - R, read Summer, 2010
    (f) Last names S - Z, read Spring, 2010

    Make a list of the *topics* the exams ask about. We will compare lists in class on Friday, February 10th.

Challenge:

1. Some second order autonomous equations can be integrated directly, often because they have ''conservation law's'', such as the conservation of energy. For example, the motion of a pendulum of length L is described in terms of it's angle $\theta(t)$ by the nonlinear second-order equation

$$\ddot{\theta} = -\frac{g}{L} sin(\theta).$$

This equation can be solved implicitly for $\theta(t)$, and is one of motivations for creating the concept of ''conservation of energy''.

   (a) Multiply both sides by $\dot{\theta}$.

   (b) Integrate both sides.

   (c) Isolate $\dot{\theta}$.

   (d) Integrate again. Your result will be something famously called an ''elliptic integral'', that mathematicians have been studying for a long time.