Homework 3, Math 251-010

Monday, January 23, 2012, due February 1, 2012

This home covers material from Sections 2.3, and 2.4, and 2.5. Refer there for more practice and help. Again, I urge you to do as many problems in the textbook as possible in addition to this assignment.

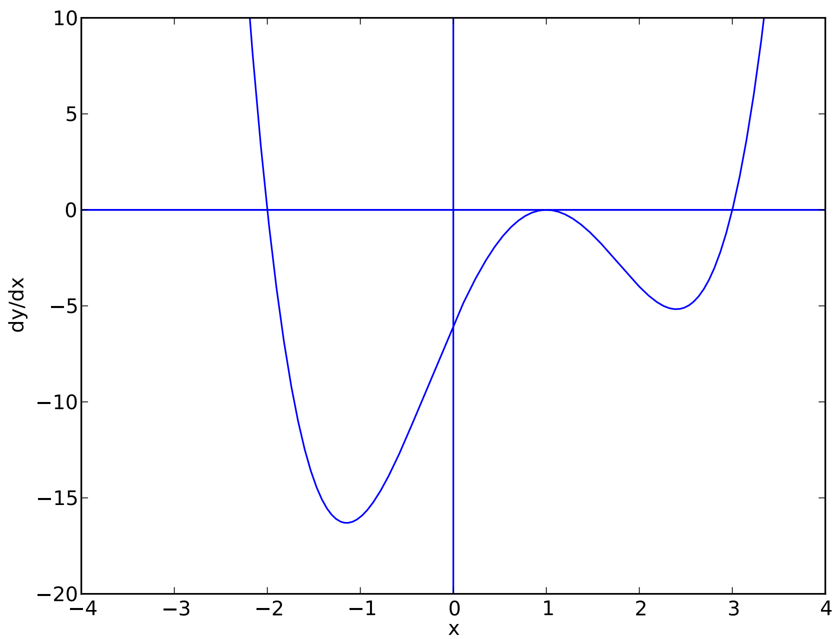
1. A tank has 100 liters of water flow in every hour. Water flows out a rate of $110 + 10\sin(2\pi t)$ liters per hour. If there are 200 liters in the tank initially, how much water is in the tank 3 hours later?

2. Particulate nitrogen is deposited in a 28,800 gallon pool at a range of $80 + 30\cos(t)$ grams per day. The pool filter pumps 10 gallons per minute, and removes 20 percent of the nitrogen passing through. If initially after filling, the pool contains 400 grams of particulate nitrogen. Determine the amount of nitrogen in the pool over time. Then determine the amount of particulate nitrogen that accumulates in the filter over time.

3. A young couple is shopping for a house. They can afford to pay \$ 900 a month (\$ 10,800 annually) in mortgage payments. If the bank agrees to give them a mortgage with a compound interest rate of 5 percent per year, how expensive would a house have to be for them to still be able to pay it off in 30 years?

4. Both $y(x) = 0$ and $y(x) = x^2$ are solutions of the linear first-order equation $xy' - 2y = 0$ passing through the point $y(0) = 0$. But Theorem 2.4.1 says there should exist one and only one solution of a linear first-order ODE. How do we reconcile this apparent discrepancy?

5. Find all the stationary solutions of $y' = y^4 - 3y^3 - 3y^2 + 11y - 6$ and determine whether they are stable, unstable, or semistable. It may help to graph the solutions.



Challenge:

1. Find an *autonomous* differential equation solved by each of the following functions.

   (a)
   $$y(x) = x^2$$

   (b)
   $$y(x) = e^{2x}$$

2. For a first order linear differential equation, $y' + p(x)y = f(x)$, the Green function for an intital-value problem is defined as

   $$G(u, v) = e^{-\int_u^v p(z)dz}.$$

   Use Leibniz's rule for differentiating integrals to show that

   $$Y(x) = y(0)G(0, x) + \int_0^x G(u, x)f(u)du$$

   is a solution of the given equations satisfying $Y(0) = y(0)$.

3. Find the Green function for $y' + ky = f(x)$.

4. Find the Green function for $xy' + ky = f(x)$.

5. Describe the asymptotic behavior of solutions of the nonlinear autonomous ODE

   $$y' = \cos y - e^y.$$