

Homework 2, Math 251-010, Spring, 2012

Assigned Wednesday, January 18, 2012
Due Wednesday, January 25, 2012

This home covers material from Sections 2.1, 2.6, and 2.3. You should work the problems at the ends of each of those sections before starting this homework.

1. Classify each of the following equations and find general or specific solutions to the equations we have studied so far.

(a)

$$(x - 3)(x + 3) = 0$$

(b)

$$y' + 10y = 3$$

(c)

$$y' + 10y = x, y(1) = 1$$

(d)

$$y' + 10y = x + \frac{1}{1+x}$$

(e)

$$\frac{dn}{dt} = 5 \frac{d^2n}{dx^2}$$

(f)

$$x \frac{\partial y(x)}{\partial x} + y(x) + e^{y(x)} \frac{\partial y(x)}{\partial x} = 0$$

(g)

$$y^3 + 3xy^2 \frac{dy}{dx} = 3 \frac{dy}{dx}$$

2. Find the range of validity for the specific solution of

$$\frac{dv}{du} = \frac{1 - 2u - 2v(u)}{1 + 2u + 2v(u)}$$

passing through $v(1) = -1$.

3. (15 points) A tank is filled with 200 liters of a solution containing 100 grams of salt. A solution containing a concentration of 2 g/liter salt enters the tank at the rate 4 liters/minute and the well-stirred mixture leaves the tank at the same rate. Set up the initial value problem for the amount of salt in the tank at time t , find the particular solution and find the limiting amount of salt in the tank as $t \rightarrow \infty$.

Challenge:

1. Special properties of 1st order linear equations.
 - (a) Solve $y' + ky = 0$, $y(0) = a$.
 - (b) Solve $y' + ky = f(x)$, $y(0) = 0$.
 - (c) Solve $y' + ky = f(x)$, $y(0) = a$.
 - (d) Find a relationship between your 3 solutions.
2. Solve $xy' + y = f(x)$, $y(0) = a$. Do the results from the first part also apply to this equation?