Homework 1, Math 251-010, Spring, 2012

Wednesday, January 11, 2012

This home covers material from Sections 1.1, 1.2, 1.3, 2.2, and 2.1. Refer there for more practice and help.

1. Classify the following differential equations as ODE or PDE[1], autonomous or non-autonomous. If the equation is an ODE, determine it's order and if the equation is separable, ratio-homogeneous, or linear. If an equation is a 1st-order ordinary differential equation and separable, ratio-homogeneous, or linear[2], find the general solution by using the appropriate integration method. DO NOT attempt to solve the other equations.

   (a)
   $$\frac{dy}{dx} = x^2 - 10\sqrt{x}$$

   (b)
   $$x^3 - \frac{dy}{dx} = 0$$

   (c)
   $$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial x^2} - 2 = 0$$

   (d)
   $$y'' + xy' - y = \sin(x)$$

   (e)
   $$\ddot{x} + 4yt\dot{x} = 0$$

   (f)
   $$3\cos(3x) + (10y - 4)y' = 0$$

   (g)
   $$\frac{dy}{dx} - \frac{y}{x} + e^{y/x} = 0$$

   (h)
   $$\frac{dx}{dt} - 5x = te^{-t}$$

   (i)
   $$\frac{1}{y'} + \frac{1}{x} = 13$$

2. Find a specific solution $v(t)$ to the equation $\dot{v} = 6 - 3v$ that passes through the initial point $v(0) = 3$.

3. Draw a direction field in $t - x$ space for the differential equation
   $$x\frac{dx}{dt} + t = 0.$$

[1] just a reminder, ODE is an acronym for Ordinary differential equation, and PDE is an acronym for Partial Differential Equation

[2] note that some linear equations are separable, but some are not.

Challenge: What does it really mean for something to be a solution of an ordinary differential equation? Here's a problem that will help you. Graphing the problem as you go may help.

Suppose we have equation $(y')^2 + 4y - 4 = 0$, where $y(x)$.

- Show that $y(x) = 1$ is a solution.
- Show that $y(x) = 1 + x^2$ is not a solution.
- Show that $y(x) = 1 - x^2$ is a solution.
- Show that $y(x) = \max\{1 - (x-1)^2, 1 - (x+1)^2\}$ is not a global solution by finding a coordinate x where the solution fails.
- Explain why

$$y(x) = \begin{cases} 1 - (x-1)^2 & \text{if } x \geq 1, \\ 1 - (x+1)^2 & \text{if } x \leq -1, \\ 1 & \text{otherwise} \end{cases}$$

  does not suffer from the problem of the previous solution.