Laplace Transform Reference Card

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Rules

$$\mathcal{L}_{sx}[f] = \int_0^\infty e^{-sx} f(x) dx = F(s)$$

$$\mathcal{L}[f'] = sF(s) - f(0)$$

$$\mathcal{L}[f''] = s^2 F(s) - sf(0) - f'(0)$$

$$\mathcal{L}[u_c(x) f(x-c)] = e^{-cs} F(s)$$

$$\mathcal{L}[e^{ax} f(x)] = F(s-a)$$

$$\mathcal{L}[f(x/a)] = aF(as)$$

$$f(x) * g(x) = \int_0^x f(x-u) g(u) du$$

$$\mathcal{L}[f(x) * g(x)] = F(s)G(s)$$

$$\mathcal{L}[(-x)^n f(x)] = \frac{d^n}{ds^n} F(s)$$

Transforms

Function $f(x)$	Laplace Transform $\mathcal{L}[f] = F(s)$
1	$\frac{1}{s}$
x^n	$\frac{n!}{s^{n+1}}$
e^{ax}	$\frac{1}{s-a}$
$x^n e^{ax}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(ax)$	$\frac{a}{s^2+a^2}$
$\cos(ax)$	$\frac{s}{s^2+a^2}$
$\sinh(ax)$	$\frac{a}{s^2-a^2}$
$\cosh(ax)$	$\frac{s}{s^2-a^2}$
$e^{ax}\sin(bx)$	$\frac{b}{(s-a)^2+b^2}$
$e^{ax}\cos(bx)$	$\frac{s-a}{(s-a)^2+b^2}$
$\delta(x-c)$	e^{-cs}
$u_c(x)$	$\frac{e^{-cs}}{s}$
$1 - \mathrm{erf}\left(\frac{a}{2\sqrt{x}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$
$\frac{1}{\sqrt{x}} e^{a^2/(4x)}$	$\sqrt{\frac{\pi}{s}} e^{-a\sqrt{s}}$
$e^{-1/4x}$	$\frac{1}{\sqrt{s}} K_1(\sqrt{s})$
$\sin(x)/x$	$\arctan(1/s)$