Math 250, Section 7, Quiz 6

Name:_____

ID Number: _____

Instructions: Clearly answer each of the questions below. Remember to check the back side. Use full sentences and proper grammer for verbal answers. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. Find the general solution of the 1st order system

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Answer:

$$\vec{x}(t) = C_1 e^{4t} \begin{bmatrix} \cos(5t) \\ \sin(5t) \end{bmatrix} + C_2 e^{4t} \begin{bmatrix} \sin(5t) \\ -\cos(5t) \end{bmatrix}$$

2. What kind of phase portraits do each of the following systems have near the stationary solution $(0, 0)$ of $\dot{x} = Ax$?

   (a) If $A = \begin{bmatrix} 2 & 3 \\ 0 & 10 \end{bmatrix}$? Answer: repelling node, unstable

   (b) If $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$? Answer: center

   (c) If the eigenvalues of A are $\lambda = -2 \pm 23i$? Answer: attracting focus, stable

   (d) If the product of the eigenvalues is negative? Answer: saddle point, unstable

3. If $x(0) = [2, -3]$, find the specific solution of the 1st order system

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Then draw the phase portrait near the steady-state solution. Answer:

$$\vec{x}(t) = \frac{5}{2} e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} - \frac{1}{2} e^{5t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$