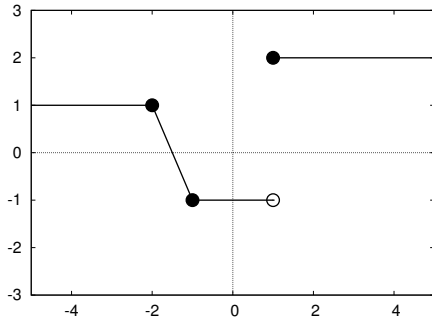


Name: _____

ID Number: _____

Instructions: Clearly answer each of the questions below. Remember to check the back side. Use full sentences and proper grammar for verbal answers. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. Use Heaviside functions $u_c(x)$ to construct the function corresponding to this graphic.



Answer:

$$f(x) = 1 - u_{-2}(x)(2x + 4) + u_{-1}(x)(2x + 2) + 3u_1(x)$$

2. Use Laplace transforms to find the specific solution of the initial value problem

$$y'' - 9y = u_4(t), \quad y(0) = 0, \quad y'(0) = 0$$

Answer:

$$y(t) = u_4(t) \frac{1}{18} (-2 + e^{3(t-4)} + e^{-3(t-4)}) = u_4(t) \frac{1}{9} [\cosh(3t - 12) - 1]$$

3. Consider the equation $y'' + 5y' + 6y = f(t)$.

(a) Find the Green function for this problem by solving $g'' + 5g' + 6g = \delta(t)$.

Answer:

$$g(t) = u_0(t) (e^{-2t} - e^{-3t})$$

(b) Use the Green function to find a particular solution of $y'' + 5y' + 6y = t$. Leave your answer as an un-evaluated integral.

Answer:

$$y_p(t) = \int_0^t (t - v) (e^{-2v} - e^{-3v}) dv$$