1. (7 pts) A mass of 3 kg is tied to the end of a spring. At rest, this mass stretches the spring 1/5th of a meter. The damping coefficient $\gamma = 6 \text{ kg/s}$, and the spring is initially pulled down 1 meter and released to set it in motion. Assume gravity’s acceleration $g = 10 \text{ m/s}^2$.

(a) Determine a differential equation with initial conditions for the displacement $u(t)$ of the mass over time.

Answer: $m = 3, k = mg/\Delta u = 150$

$$3u'' + 6u' + 150u = 0, \quad u(0) = 1, \quad u'(0) = 0.$$

(b) Find the specific solution $u(t)$ of this equation.

Answer: $u(t) = e^{-t}(C_1 \sin(7t) + C_2 \cos(7t))$

(c) Determine the quasi-period of oscillations of the mass.

Answer: The period $T = \frac{7}{2\pi}$
2. (8 pts) Consider the general series solution \( y(x) = \sum_{n=0}^{\infty} a_n x^n \) about \( x_0 = 0 \) to the variable-coefficient ODE
\[
y'' - 2xy' + 6y = 0.
\]
(a) Find the recurrence equation for \( a_n \) in terms of lower-order terms.
Answer:
\[
a_n = \frac{2(n-5)a_{n-2}}{n(n-1)}
\]
(b) Calculate \( a_2..a_6 \) in terms of \( a_0 \) and \( a_1 \).
Answer:
\[
\begin{align*}
a_2 &= -3a_0, \\
a_3 &= (-2/3)a_1, \\
a_4 &= (-1/6)a_2 = (-1/6)(-3)a_0 = (1/2)a_0, \\
a_5 &= 0, \\
a_6 &= (1/15)a_4 = (1/15)(1/2)a_0 = (1/30)a_0, \ldots
\end{align*}
\]
(c) Separate this solution into its linearly-independent parts \( y_1(x) \) and \( y_2(x) \).
Answer:
\[
C_1 \left( \frac{1}{30}x^6 + \frac{1}{2}x^4 - 3x^2 + 1 \right) + C_2 \left( -\frac{2}{3}x^3 + x \right)
\]
(d) Write down the general solution for \( y(x) \).