Math 250, Section 7, Quiz 4

Name:________________________________

ID Number: ________________________________

Instructions: Clearly answer each of the questions below. Remember to check the back side. Use full sentences and proper grammer for verbal answers. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. (7 pts) A mass of 3 kg is tied to the end of a spring. At rest, this mass stretches the spring 1/5th of a meter. The damping coefficient $\gamma = 6$ kg/s, and the spring is initially pulled down 1 meter and released to set it in motion. Assume gravity's acceleration $g = 10m/s^2$.

   (a) Determine a differential equation with initial conditions for the displacement $u(t)$ of the mass over time.

   Answer: $m = 3$, $k = mg/\Delta u = 150$

   $$3u'' + 6u' + 150u = 0, \quad u(0) = 1,\ u'(0) = 0.$$

   (b) Find the specific solution $u(t)$ of this equation.

   Answer:

   $$u(t) = e^{-t}\left(C_1 \sin(7t) + C_2 \cos(7t)\right)$$

   (c) Determine the quasi-period of oscillations of the mass.

   Answer: The period $T = \dfrac{7}{2\pi}$

2. (8 pts) Consider the general series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ about $x_0 = 0$ to the variable-coefficient ODE

$$y'' - 2xy' + 6y = 0.$$

(a) Find the recurrence equation for a_n in terms of lower-order terms.

Answer:

$$a_n = \frac{2(n-5)a_{n-2}}{n(n-1)}$$

(b) Calculate $a_2..a_6$ in terms of a_0 and a_1.

Answer:

$$\begin{aligned} a_2 &= -3a_0, \\ a_3 &= (-2/3)a_1, \\ a_4 &= (-1/6)a_2 = (-1/6)(-3)a_0 = (1/2)a_0, \\ a_5 &= 0, \\ a_6 &= (1/15)a_4 = (1/15)(1/2)a_0 = (1/30)a_0, \dots \end{aligned}$$

(c) Separate this solution into it's linearly-independent parts $y_1(x)$ and $y_2(x)$.

Answer:

$$C_1\left(\frac{1}{30}x^6 + \frac{1}{2}x^4 - 3x^2 + 1\right) + C_2\left(-\frac{2}{3}x^3 + x\right)$$

(d) Write down the general solution for $y(x)$.