Math 250, Section 7, Quiz 3

Name:____________________________________

ID Number: ____________________________________

Instructions: Clearly answer each of the questions below. Remember to check the back side. Use full sentences and proper grammer for verbal answers. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. (a) Find a particular solution of the inhomogeneous equation $\ddot{y} + 2\dot{y} - 8y = t$.

   Answer: Try $y(t) = At + B$. On substitution, we find $0 + 2A - 8(At + B) = t$, so $-8At = t$ and $2A - 8B = 0$. This leads to $y(t) = -t/8 - 1/32$ as one particular solution.

   (b) If $y_p(t) = \dfrac{e^{3t}}{7}$ is a particular solution of the inhomogeneous equation $\ddot{y} + 2\dot{y} - 8y = e^{3t}$, find a particular solution of the inhomogeneous equation $\ddot{y} + 2\dot{y} - 8y = -8t + 4e^{3t}$.

   Answer: The principle of super-position says that we can add solutions of linear equations in a certain way. In this case, if $y_1(t) = \dfrac{e^{3t}}{7}$ solves $\ddot{y} + 2\dot{y} - 8y = e^{3t}$, and $y_2(t) = -t/8 - 1/32$ solves $\ddot{y} + 2\dot{y} - 8y = t$, then a solution of $\ddot{y} + 2\dot{y} - 8y = -8t + 4e^{3t}$ is

$$-8y_2 + 4y_1 = t + 1/4 + \frac{4}{7}e^{3t}.$$

2. At what points can unique solutions to the 3rd order equation $(t^2 - 9)y''' + (t^2 - 4)y'' + t^2 y = \frac{1}{t}$ not be found?

   Answer: In standard form, this 3rd order linear equation is re-written as

$$(t^2 - 9)y''' + \frac{(t^2 - 4)}{t^2 - 9}y'' + \frac{t^2}{t^2 - 9}y = \frac{1}{t(t^2 - 9)}$$

   The coefficients are not continuous at $t \in \pm 3, 0$, so those are the points where we can not find a unique solution.

3. Find the general solution of the 4th-order homogeneous equation $y'''' - 13y'' + 36y = 0$.

Answer: The characteristic equation $r^4 - 13r^2 + 36 = 0$ is quartic, but is actually easy to factor.

$$(r^2 - 9)(r^2 - 4) = 0.$$

Inspecting this equation, we see $r \in \pm 2, \pm 3$, so the general solution

$$y(t) = C_1 e^{2t} + C_2 e^{-2t} + C_3 e^{3t} + C_4 e^{-3t}$$

4. Classify each of the following sets of solutions of a linear differential equation as linearly dependent or linearly independent. (Remember, only linearly independent sets can be fundamental solution sets)

(a) ________________ $\{e^{2t}, e^{-2t}\}$ Answer: Linearly independent

(b) ________________ $\{e^{10t}, \sin(7t), \cos(7t)\}$ Answer: Linearly independent

(c) ________________ $\{\sin(2t), \cos(2t), \sin(3-2t)\}$ Answer: Linearly dependent because $\sin(3-2t) = \sin(3)\cos(2t) - \sin(2t)\cos(3)$, the weighted sum of the first two functions.

(d) ________________ $\{1+t^2, t+t^2, 3+3t+6t^2\}$ Answer: Linearly dependent, because the third is the sum of the first two --- $3(1+t^2) + 3(t+t^2) = 3 + 3t + 6t^2$