Math 250, Section 7, Quiz 2

Name:______________________________

ID Number: ______________________________

Instructions: Clearly answer each of the questions below. Remember to check the back side. Use full sentences and proper grammer for verbal answers. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. Classify the following equation, and determine the largest interval over which a solution through the point $y(1) = 2$ can be uniquely constructed.

$$(\sin x)y + (x^2 - 9)y' = e^{37x}$$

Answer: This is a 1st-order linear ordinary differential equation, non-autonomous, not separable. In standard form,

$$y' + \frac{\sin x}{x^2 - 9} y = e^{37x} x^2 - 9$$

There are singularities at $x = 3$ and $x = -3$, so the largest interval containing $x = 1$ where we can construct the solution is $(-3, 3)$.

2. Determine the stability of all stead-state solutions of the first-order autonomous ordinary differential equation

$$2\frac{dx}{dt} = 30x^2 - 5x^3.$$

Answer: Factoring and transforming to a more familiar form, $\dot{x} = (5/2)x^2(6 - x)$.. The stationary solutions are $x = 6$ and $x = 0$. Testing points, we find $x > 6$ implies $\dot{x} < 0$, $x < 0$ implies $\dot{x} > 0$, and $0 < x < 6$ implies $\dot{x} > 0$. So $x = 0$ is semistable, and $x = 6$ is stable.

3. What special properties (discussed in Section 3.2) do linear equations have that make them easier to solve than nonlinear equations?

Answer: The solutions of linear equations can be constructed by adding different solutions together, thanks to the super-position principle. Specifically, if

$$y_1'' + p(x)y_1' + q(x)y_1 = g_1(x),$$

$$y_2'' + p(x)y_2' + q(x)y_2 = g_2(x)$$

then $y = y_1 + y_2$ solves

$$y'' + p(x)y' + q(x)y = g_1(x) + g_2(x)$$

4. Find the general solution of the second-order equation $y'' - 2y + 2y' = 0$.

Answer: First, we re-arrange the equation so it is in standard form. $y'' + 2y' - 2y = 0$. Now, the characteristic equation is $r^2 + 2r - 2 = 0$. Then using the quadratic formula, $r \in -1 \pm \sqrt{3}$. We've found two distinct real roots, so we know the Wronskian will be non-zero. Thus, the general solution is

$$y(x) = C_1 e^{(-1+\sqrt{3})x} + C_2 e^{(-1-\sqrt{3})x}$$

5. Calculate the Wronskian for the solution set $\{e^{2t}, te^{2t}\}$, and use it to determine if this set is a fundamental solution set.

Answer: We let $y_1(t) = e^{2t}$ and $y_2(t) = te^{2t}$. The general form for the Wronskian is $W = y_1 y_2' - y_2 y_1'$.

$$W = e^{2t}(e^{2t} + 2te^{2t}) - te^{2t}(2e^{2t}) = e^{4t}$$

Since $W \neq 0$, this is a fundamental solution set (of a second-order linear equation).