

Name: _____

ID Number: _____

Instructions: Clearly answer each of the questions below. Remember to check the back side. Use full sentences and proper grammar for verbal answers. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. (3 pts) Select the letter of the correct direction field for each first order ordinary differential equation.

(a) _____ $y' = 2\frac{x}{y}$

Answer: D

(b) _____ $y' = -2\frac{y}{x}$

Answer: F

(c) _____ $y' = -2x + y$

Answer: E

(d) _____ $y' = x^2 + 2x$

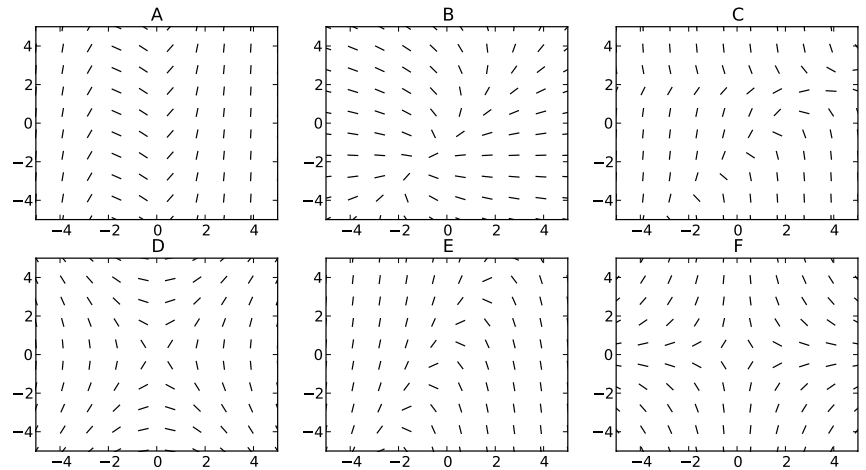
Answer: A

(e) _____ $y' = (y - 2)(x - y - 2)$

Answer: C

(f) _____ $y' = \frac{y+2}{2x-y}$

Answer: B



2. (3 pts) Consider the differential equation

$$\sqrt{\psi + ke^m + \frac{d^3\psi}{dm^3}} = \left(\frac{d\psi}{dm}\right)^5 \sin(m) - \pi.$$

(a) This equation is _____ differential equation. Answer: Ordinary

(b) It is a _____ order equation. Answer: 3rd order

(c) The independent variable is _____. Answer: m

(d) The dependent variable is _____. Answer: ψ

(e) Is the equation autonomous? _____. Answer: no, it is non-autonomous

3. (3 pts) Circle each that is a solution of the equation $y' = y^2$.

a. 0 b. 1 c. 2x

d. $\frac{1}{2-x}$ e. $\frac{-1}{x}$ f. e^x

Answer: a,d,and e are solutions

4. (3 pts) Find the general form of solutions to the equation $y' = 3 + 2y$.

Answer: The equation is both separable and linear. If we treat it as linear, $y' - 2y = 3$,

$$p(x) = -2, \quad g(x) = 3,$$

$$\mu(x) = e^{\int -2dx} = e^{-2x}$$

$$y(x) = \frac{1}{\mu(x)} \left(C + \int g(x)\mu(x)dx \right)$$

$$y(x) = e^{2x} \left(C + \int 3e^{-2x}dx \right)$$

$$y(x) = Ce^{2x} - \frac{3}{2}$$

5. (3 pts) Describe in English the meaning of the falling body equation $m \frac{dv}{dt} = mg - \gamma v$.

Answer: This equation is derived from Newton's second law, that the acceleration of an object (dv/dt) times the mass of the object (m) is equal to the sum of the forces on the object. Here, the force of gravity is represented by the mass m times gravities acceleration g , and the force of drag on the object is proportional to the magnitude of the velocity v , but in the opposite direction, with drag coefficient γ .