This exam has 11 questions for a total of 100 points. There are 6 Multiple Choice questions and 5 Partial Credit questions. Clearly indicate your choice for each multiple choice question in the space provided for your Answer. In order to obtain full credit for partial credit problems, all work must be shown. Check that your exam has all 11 questions.

You may not use a calculator on this exam.
You may not use a cell phone on this exam.
You may not use any notes or books on this exam.
This exam booklet will be collected at the end of the exam.

Do not write in this box.

1: __________
2: __________
3: __________
4: __________
5: __________
6: __________
7: __________
8: __________
9: __________
10: __________
11: __________
Total: __________
1. (5 points) For a certain spring - mass system which satisfies the ODE:

\[ 2u'' + 2u' + 5u = 0, \]

which of the following statements is/are true?

I) The system is underdamped.
II) The system is overdamped.
III) The solution will oscillate, and its limit is 0.
IV) The solution will oscillate, and its limit is does not exist.

(a) I and IV only.
(b) II and IV only.
(c) I and III only.
(d) II and III only.

Answer: ________

2. (5 points) For the ODE \( y'' - 4y' + 4y = te^{2t} \), which is the best assumption for the form of the particular (specific) solution \( y_p \) ?

(a) \( Ate^{2t} + Be^{-2t} \).
(b) \( At^2e^{2t} + Bte^{2t} \).
(c) \( At^3e^{2t} \).
(d) \( At^3e^{2t} + Bt^2e^{2t} \).

Answer: ________
3. (5 points) What is the form of the general solution to the ODE whose characteristic equation factors as:

\[(r - 3)(r + 1)(r - (1 + i))^2(r - (1 - i))^2\]

(a) \(C_1 e^{3t} + C_2 e^{-t} + C_3 e^t \cos t + C_4 e^t \sin t + C_5 e^t \cos(-t) + C_6 t e^t \sin(-t)\).

(b) \(C_1 e^{3t} + C_2 e^{-t} + C_3 e^t \cos t + C_4 e^t \sin t + C_5 t e^t \cos t + C_6 t e^t \sin t\).

(c) \(C_1 e^{-3t} + C_2 e^t + C_3 e^t \cos t + C_4 e^t \sin t + C_5 t e^t \cos t + C_6 t e^t \sin t\).

(d) None of the above.

Answer:_______

4. (5 points) Which of the following represents the sum below, but rewritten as a sum with generic term \(t^n\)?

\[\sum_{n=0}^{\infty} (n + 1)a_{n+1} t^{n+2}\]

(a) \(\sum_{n=0}^{\infty} (n - 1)a_{n+1} t^n\).

(b) \(\sum_{n=2}^{\infty} (n + 1)a_{n+1} t^n\).

(c) \(\sum_{n=0}^{\infty} (n + 1)a_{n+1} t^n\).

(d) \(\sum_{n=2}^{\infty} (n - 1)a_{n+1} t^n\).

Answer:_______
5. (5 points) Consider the ODE:

\[(\cos x)(x - \pi/3)y'' + 2y' - 12(\cos 4x)y = 0.\]

Which is the best lower bound for the radius of convergence of the series solution about \(x_0 = 0\)?

(a) \(\pi/8\).
(b) \(\pi/3\).
(c) \(\pi/2\).
(d) None of the above.

**Answer:**

6. (5 points) Which is the Laplace transform of

\[g(t) = \begin{cases} 
1 & \text{if } t < 2 \\
t + 2 & \text{if } 2 \leq t
\end{cases}
\]

(a) \(1/s + e^{-2s}(1/s^2 + 2/s)\).
(b) \(1/s + e^{-2s}(1/s^2 + 3/s)\).
(c) \(1/s + e^{-2s}/s^2\).
(d) \(1/s + e^{-2s}(1/s^2 + 4/s)\).

**Answer:**
7. (14 points) Consider the ODE

\[ y'' - xy' - 2y = 0. \]

Find a power series solution about \( x_0 = 0 \) in the following steps:

(a) Write the recurrence relation.

(b) Compute the first 6 coefficients in terms of \( a_0 \) and \( a_1 \).

(c) Determine a formula for the coefficients \( a_n \).
8. (14 points) Determine the inverse Laplace transform of:

(a) 
\[ Y(s) = \frac{3}{s^2 + 4s + 7} \]

(b) 
\[ Y(s) = \frac{5e^{-2s}}{s - 2} + \frac{e^{-s} + 1}{s} \]
9. (14 points) Use the method of undetermined coefficients to solve the IVP.

\[ y'' - 3y' + 2y = 2 \sin t \quad y(0) = 5, \ y'(0) = 4 \]

You must use the method of undetermined coefficients to receive full credit.
10. (14 points) Solve the IVP using Laplace transforms.

\[ y'' - 6y' + 9y = u_4(t) \quad y(0) = 0, \ y'(0) = 0 \]
11. (14 points) Find a power series solution about $x_0 = 0$ for the ODE in the steps listed below.

$$y'' - y = 0.$$ 

(a) Write the recurrence relation.

(b) Compute the first 6 coefficients in terms of $a_0$ and $a_1$.

(c) Determine a formula for the coefficients $a_n$. (If these power series are familiar to you, write the common names of the functions they represent for extra credit.)