# Homework 4, Math 250-07
## due Wednesday, September 26, 2012

This home covers material from Sections 3.2, 3.3, and 3.4. Refer there for more practice and help. I urge you to do as many problems in the textbook as possible in addition to this assignment.

1. Section 3.2 of our textbook, problem 5. Answer: *The Wronskian* $W = -e^{2t}$.
2. Section 3.2 of our textbook, problem 9. Answer: *If we start at* $t = 3$*, as given, then we can build a solution to any* $t \in (0, 4)$.
3. Section 3.2 of our textbook, problem 25. Answer: *The Wronskian* $W = e^{2t}$*, so this is a fundamental solution set.*
4. Find the general form of the Wronskian $W$ of the solution set $\{e^{rt}\cos(wt), e^{rt}\sin(wt)\}$, and determine for which values of $r$ and $w$ this is in-fact a fundamental solution set (It's a fundamental solution set when $W \neq 0$). Answer: *If we let* $x_1(t) = e^{rt}\cos(wt)$ *and* $x_2(t) = e^{rt}\sin(wt)$*, then*

$$W = x_1\dot{x}_2 - x_2\dot{x}_1 = we^{2rt}.$$

$W \neq 0$ *if and only if* $w \neq= 0$. *The value of* $r$ *does not matter.*

5. Consider the differential equation

$$365\,\mathrm{x}(t) + 38\frac{\partial}{\partial t}\mathrm{x}(t) + \frac{\partial^2}{\partial^2 t}\mathrm{x}(t) = 0.$$

(a) Find the characteristic equation. Answer:

$$r^2 + 38r + 365 = 0$$

(b) Find a fundamental solution set. Answer:

$$\left\{e^{-19t}\cos(2t), e^{-19t}\sin(2t)\right\}$$

(c) Calculate the Wronskian and show that it is non-zero. Answer:

$$W = \left(-19e^{-19t}\sin(2t) + 2e^{-19t}\cos(2t)\right)e^{-19t}\cos(2t) - \left(-2e^{-19t}\sin(2t) - 19e^{-19t}\cos(2t)\right)e^{-19t}\sin(2t)$$

$$W = 2e^{-38t}$$

(d) Construct the general solution. Answer:

$$x(t) = C_1e^{-19t}\cos(2t) + C_2e^{-19t}\sin(2t)$$

6. Consider the differential equation

$$234\,\mathrm{x}(t) + 30\frac{\partial}{\partial t}\mathrm{x}(t) + \frac{\partial^2}{\partial^2 t}\mathrm{x}(t) = 0.$$

(a) Find the characteristic equation. Answer:

$$r^2 + 30r + 234 = 0$$

(b) Find a fundamental solution set. Answer:

$$\left\{e^{-15t}\cos(3t), e^{-15t}\sin(3t)\right\}$$

(c) Calculate the Wronskian and show that it is non-zero. Answer:

$$W = \left(-15e^{-15t}\sin(3t) + 3e^{-15t}\cos(3t)\right)e^{-15t}\cos(3t) - \left(-3e^{-15t}\sin(3t) - 15e^{-15t}\cos(3t)\right)e^{-15t}\sin(3t)$$

$$W = 3e^{-30t}$$

(d) Construct the general solution. Answer:

$$x(t) = C_1 e^{-15t} \cos(3t) + C_2 e^{-15t} \sin(3t)$$

(e) Find the specific solution satisfying initial conditions $x(0) = -3$, $\dot{x}(0) = 42$. Answer:

$$x(t) = (-3)e^{-15t} \cos(3t) + (-1)e^{-15t} \sin(3t)$$

7. Consider the differential equation

$$4\,\mathrm{x}(t) + 4\frac{\partial}{\partial t}\,\mathrm{x}(t) + \frac{\partial^2}{\partial^2 t}\,\mathrm{x}(t) = 0.$$

(a) Find the characteristic equation. Answer:

$$r^2 + 4r + 4 = 0$$

(b) Find a fundamental solution set. Answer:

$$\left\{e^{-2t}, te^{-2t}\right\}$$

(c) Calculate the Wronskian and show that it is non-zero. Answer:

$$W = 2te^{-4t} + \left(-2te^{-2t} + e^{-2t}\right)e^{-2t}$$
$$W = e^{-4t}$$

(d) Construct the general solution. Answer:

$$x(t) = C_1 e^{-2t} + C_2 te^{-2t}$$

8. Consider the second-order homogeneous equation with variable coefficients

$$(3x-1)\frac{\partial^2}{\partial^2 x}\,\mathrm{y}(x) + \left(18x^2 - 5\right)\frac{\partial}{\partial x}\,\mathrm{y}(x) + \left(36x^2 - 12x - 6\right)\mathrm{y}(x) = 0.$$

(a) Use direct substitution to find the value of $r$ for which $e^{rt}$ is a solution to this equation. (Note: The factorization may require persistence.) Answer:

$$r^2(3x-1)e^{rx} + r\left(18x^2 - 5\right)e^{rx} + \left(36x^2 - 12x - 6\right)e^{rx} = 0$$
$$(r+2)\left(3rx - r + 18x^2 - 6x - 3\right)e^{rx} = 0$$
$$r = -2$$

(b) Use reduction-of-order to construct the general solution of this equation. Answer:

$$\mathrm{y}(x) = \mathrm{Z}(x)e^{-2x}$$
$$18x^2 e^{-2x}\frac{\partial}{\partial x}\mathrm{Z}(x) - 12xe^{-2x}\frac{\partial}{\partial x}\mathrm{Z}(x) + 3xe^{-2x}\frac{\partial^2}{\partial^2 x}\mathrm{Z}(x) - e^{-2x}\frac{\partial}{\partial x}\mathrm{Z}(x) - e^{-2x}\frac{\partial^2}{\partial^2 x}\mathrm{Z}(x)$$
$$\left(3xe^{-2x} - e^{-2x}\right)\frac{\partial^2}{\partial^2 x}\mathrm{Z}(x) + \left(18x^2 e^{-2x} - 12xe^{-2x} - e^{-2x}\right)\frac{\partial}{\partial x}\mathrm{Z}(x)$$
$$\mathrm{Y}(x) = \frac{\partial}{\partial x}\mathrm{Z}(x)$$
$$\left(3xe^{-2x} - e^{-2x}\right)\frac{\partial}{\partial x}\mathrm{Y}(x) + \left(18x^2 e^{-2x} - 12xe^{-2x} - e^{-2x}\right)\mathrm{Y}(x)$$
$$\mathrm{Y}(x) = C_2\left(x - \frac{1}{3}\right)e^{x(-3x+2)}$$
$$C_2\left(x - \frac{1}{3}\right)e^{x(-3x+2)} - \frac{\partial}{\partial x}\mathrm{Z}(x)$$
$$\mathrm{Z}(x) = C_1 + C_2 e^{x(-3x+2)}$$
$$\mathrm{y}(x) = C_1 e^{-2x} + C_2 e^{-3x^2}$$