Math 220, Quiz 9 Section 1

Name:_____

ID Number:_____

Instructions: Clearly answer each of the questions below. Show your work and any formulas you employ. You can leave square roots unsimplified.

For the problems below, let the matrix

$$M = \begin{bmatrix} -6 & 3 & -2 \\ 3 & 2 & -6 \\ -2 & -6 & -3 \end{bmatrix}.$$

1. Find a basis of the eigenspace for eigenvalue $\lambda = -7$.

$$\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

2. Find an orthogonal basis of the eigenspace for eigenvalue $\lambda = -7$.

$$\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{5} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \right\}$$

3. Find an orthonormal basis of the eigenspace for eigenvalue $\lambda = -7$.

$$\left\{ \frac{1}{\sqrt{10}} \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{35}} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \right\}$$

4. If $\lambda = 7$ is also an eigenvalue of M with corresponding unit eigenvector $\frac{1}{\sqrt{14}}[-1, -3, 2]^T$, what pair of matrices D and P provide an orthogonal diagonalization of $M = PDP^T$?

$$D = \begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$P = \begin{bmatrix} \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{35}} & \frac{-1}{\sqrt{14}} \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{35}} & \frac{-3}{\sqrt{14}} \\ 0 & \frac{5}{\sqrt{35}} & \frac{2}{\sqrt{14}} \end{bmatrix}$$