

Instructions: Clearly answer each of the questions below. Remember to check the back side. Show your work and any formulas you employ. Simplify all answers as far as possible.

1. (2 pts) What algorithm did we learn for making an orthogonal basis out of a spanning set of vectors? The Gram–Schmidt process

2. (6 pts) Consider the vector  $\mathbf{y} = [5, -4, -1]$  and the vector  $\mathbf{u} = [1, 0, -1]$ .

(a) Find the orthogonal projection  $\mathbf{y}_{\parallel}$  of  $\mathbf{y}$  in the direction of  $\mathbf{u}$ .

$$\begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$$


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(b) Find the residual vector  $\mathbf{y}_{\perp}$  of  $\mathbf{y}$  orthogonal  $\mathbf{u}$ .

$$\begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix}$$


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(c) Give an orthogonal basis of the subspace spanned by  $\mathbf{y}$  and  $\mathbf{u}$ .

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix} \right\}$$


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Any rescaling of these two vectors also forms an orthogonal basis of the span of  $\mathbf{y}$  and  $\mathbf{u}$ . We have essentially just done one step of the Gram–Schmidt process, with  $\mathbf{u}$  being our first vector and  $\mathbf{y}_{\perp}$  being our second.