

Instructions: Clearly answer each of the questions below. Remember to check the back side. Show your work and any formulas you employ. Simplify all answers as far as possible.

1. (1 pt) The nullspace of  $A$  is the set of all solutions to what equation?  $A\vec{x} = \vec{0}$

2. (1 pt) What vector is always an element of the nullspace of  $A$ , no matter what the matrix  $A$  is?  $\vec{0}$

3. (2 pt) Calculate the following matrix-vector product.

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 5 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} \quad \underline{\begin{bmatrix} 5 \\ -9 \\ 10 \end{bmatrix}}$$

4. (2 pt) What is the nullspace of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} ? \quad \underline{\vec{x} = x_4 \begin{bmatrix} -1 \\ -3 \\ 1 \\ 1 \end{bmatrix}}$$

5. (2 pt) What is the general solution of  $A\mathbf{x} = \mathbf{b}$  if  $\text{Null}(A) = \left\{ c_1 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \end{bmatrix} : (c_1, c_2) \in \mathbb{R}^2 \right\}$

and  $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$  is a solution of  $A\mathbf{x} = \mathbf{b}$ ?

$$\underline{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \end{bmatrix}}$$