Math 220, Quiz 8 Section 8 Name:____________________

ID Number:____________________

Instructions: Clearly answer each of the questions below. Remember to check the back side. Show your work and any formulas you employ. Simplify all answers as far as possible.

1. (2 pts) Find values for a and b so that the matrix below is symmetric.

$$\begin{bmatrix} 0 & a & 6 \\ 8 & 12 & 10 \\ b & 10 & 2 \end{bmatrix}$$

$a = 8$

$b = 6$

2. (6 pts) Use the Gram–Schmidt process to find an orthogonal basis of the subspace of $\mathbb{R}^4$ spanned by the set of vectors

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 6 \\ 5 \end{bmatrix} \right\}$$

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 3 \\ 2 \end{bmatrix} \right\}$$

Note: When you use the Gram–Schmidt algorithm, you have to use your accumulating orthogonal vectors as you go, not your original vectors – otherwise, you might end up "double-counting" parts of the space, and your vectors won't be orthogonal. So, given basis, $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, an orthogonal basis spanning the same subspace is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ with

$$\begin{aligned} \mathbf{v}_1 &= \mathbf{u}_1 \\ \mathbf{v}_2 &= \mathbf{u}_2 - \frac{\mathbf{v}_1 \cdot \mathbf{u}_2}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 \\ \mathbf{v}_3 &= \mathbf{u}_3 - \frac{\mathbf{v}_1 \cdot \mathbf{u}_3}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 - \frac{\mathbf{v}_2 \cdot \mathbf{u}_3}{\mathbf{v}_2 \cdot \mathbf{v}_2}\mathbf{v}_2. \end{aligned}$$

or if you're like me and like to use only matrix products,

$$\begin{aligned} \mathbf{v}_1 &= \mathbf{u}_1 \\ \mathbf{v}_2 &= \mathbf{u}_2 - \frac{\mathbf{v}_1\mathbf{v}_1^T\mathbf{u}_2}{\mathbf{v}_1^T\mathbf{v}_1} \\ \mathbf{v}_3 &= \mathbf{u}_3 - \frac{\mathbf{v}_1\mathbf{v}_1^T\mathbf{u}_3}{\mathbf{v}_1^T\mathbf{v}_1} - \frac{\mathbf{v}_2\mathbf{v}_2^T\mathbf{u}_3}{\mathbf{v}_2^T\mathbf{v}_2}. \end{aligned}$$