Math 220, Quiz 7 Section 8

Name:________________________________

ID Number:________________________

Instructions: Clearly answer each of the questions below. Remember to check the back side. Show your work and any formulas you employ. Simplify all answers as far as possible.

1. (4 pts) Consider the orthogonal basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} \right\}$ of a subspace S of $\mathbb{R}^4$.

   (a) If the vector $\mathbf{x}$ is orthogonal to both these vectors, then $\mathbf{x}$ is in the nullspace of what matrix? $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -3 & 1 & 0 \end{bmatrix}$

   Two vectors u and x are orthogonal if their dot-product is 0, which is the same as saying $u^T x = 0$.

   (b) What is the dimension of the orthogonal complement $S^{\perp}$? 2

   A subspace plus its orthogonal complement is always the full space, so since $\dim(S) = 2$, $\dim(S^{\perp}) = 4 - 2 = 2$.

   (c) Find a basis of the orthogonal complement $S^{\perp}$. The orthogonal complement $S^{\perp}$ is the set of all vectors orthogonal to our given basis and S, so $S^{\perp}$ is equal to the nullspace of our answer to 1a. by calculating a basis of this nullspace using row-reduction, we get our answer. I've multiplied the answers basis vectors by 5 to eliminate fractions – they still form a basis of the same space.

   $$\left\{ \begin{bmatrix} -4 \\ -1 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \\ 5 \end{bmatrix} \right\}$$

2. (4 pts) Rewrite the vector $\mathbf{y} = [4, 5]$ as the sum of two vectors parallel and perpendicular to the vector $\mathbf{u} = [-1, 2]$.

   $$\mathbf{y}_{\parallel} = \begin{bmatrix} -6/5 \\ 12/5 \end{bmatrix}$$

   $$\mathbf{y}_{\perp} = \begin{bmatrix} 26/5 \\ 13/5 \end{bmatrix}$$