

Instructions: Clearly answer each of the questions below. Remember to check the back side. Show your work and any formulas you employ. Simplify all answers as far as possible.

- (1 pt) If a set of vectors is linearly dependent, then there is a _____ non-trivial linear combination of the vectors that vanishes.
- (1 pt) If the solution set of the homogeneous linear system $A\mathbf{x} = \mathbf{0}$ involves _____ linearly dependent one or more free variables, then the set of column vectors of A is _____
- (6 pt) For each of the sets of vectors below, is the *linearly independent* or *linearly dependent*?

(a) $\left\{ \begin{bmatrix} 2 \\ -6 \\ -5 \end{bmatrix} \right\}$ linearly independent

(b) $\left\{ \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \right\}$ linearly dependent

(c) $\left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ linearly dependent

(d) $\left\{ \begin{bmatrix} 3 \\ 9 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \\ 13 \end{bmatrix}, \begin{bmatrix} 6 \\ -9 \\ 4 \end{bmatrix} \right\}$ linearly independent

(e) $\left\{ \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \\ 11 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix} \right\}$ linearly dependent

(f) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ linearly dependent

Only (d) and (e) require you to resort to calculation to determine linear dependence or independence. All the others can be answered at a glance. A single non-zero vector is always independent. If two vectors are scalar multiples of each other, they are dependent. Any set of vectors containing the zero vector is linearly dependent. And any set of vectors where the number of vectors exceeds the number of entries in the vectors is linearly dependent.