Math 220, Quiz 8 Section 1

Name:_____

ID Number:_____

Instructions: Clearly answer each of the questions below. Remember to check the back side. Show your work and any formulas you employ. Simplify all answers as far as possible.

1. (1 pt) Does the outer product of two vectors create a number, a vector, or a matrix?

   A matrix

2. (3 pts) If $T(\mathbf{x}) = P\mathbf{x}$ is a linear transformation that returns the orthogonal projection of any $\mathbb{R}^3$ vector $\mathbf{x}$ in the direction of the vector $[1, -3, 1]$, what is the matrix P?

   $$\frac{1}{11}\begin{bmatrix} 1 & -3 & 1 \\ -3 & 9 & -3 \\ 1 & -3 & 1 \end{bmatrix}$$

   Letting $\mathbf{u} = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$, the matrix for the orthogonal projection is

   $$P = \frac{\mathbf{u}\mathbf{u}^T}{\mathbf{u}^T\mathbf{u}} = \frac{\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -3 & 1 \end{bmatrix}}{\begin{bmatrix} 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}} = \frac{\begin{bmatrix} 1 & -3 & 1 \\ -3 & 9 & -3 \\ 1 & -3 & 1 \end{bmatrix}}{11}$$

3. (4 pts) Consider the point $\mathbf{y} = [-4, 16]$ and the line passing through the origin and the point $\mathbf{u} = [2, 5]$.

   (a) Find the orthogonal projection $\mathbf{y}_{\parallel}$ of $\mathbf{y}$ onto the line through $\mathbf{u}$.

   $$\mathbf{y}_{\parallel} = \frac{1}{29}\begin{bmatrix} 144 \\ 360 \end{bmatrix}$$

   (b) What is distance from the line to $\mathbf{y}$?

   $$\sqrt{\frac{2704}{29}} = \frac{52}{\sqrt{29}} \approx 9.66$$

   There was a typo in this problem – the calculation is too length and cumbersome for easy hand-calculation. The problems should have had $\mathbf{y} = [-11, 16]$. In this case, you would have found $\mathbf{y}_{\parallel} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$ and the distance would be $||\mathbf{y}_{\perp}|| = ||\mathbf{y} - \mathbf{y}_{\parallel}|| = \sqrt{261} = 3\sqrt{29}$.