

Instructions: Clearly answer each of the questions below. Remember to check the back side. Show your work and any formulas you employ. Simplify all answers as far as possible.

1. (4 pts) The matrix $N = \begin{bmatrix} 0 & 5 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ is called a nilpotent matrix.

- (a) What is the algebraic multiplicity of the eigenvalue $\lambda = 0$?
 (b) What is the geometric multiplicity of the eigenvalue $\lambda = 0$?
 (c) Is the matrix diagonalizable?
 (d) Calculate N^3 .

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 No, $3 \neq 1$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. (4 pts) Calculate an invertible matrix P and a diagonal matrix D that diagonalize $A = \begin{bmatrix} 5 & 4 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$.

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$
