

Instructions: Clearly answer each of the questions below. Remember to check the back side. Show your work and any formulas you employ. Simplify all answers as far as possible.

1. (2 pts) What is the nullspace of the matrix  $A$  below?

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\left\{ x_3 \begin{bmatrix} -5 \\ 6 \\ 1 \end{bmatrix} : x_3 \in \mathbb{R} \right\}$$

2. (1 pt) A linear transformation is a function for which  $T(c\mathbf{x}) = cT(\mathbf{x})$  and  $T(\mathbf{x} + \mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y})$

3. (1 pt) Is the function  $T(x) = 4 - x$  a linear transformation?

\_\_\_\_\_ No

4. (1 pt) Is the function  $T(x) = -x$  a linear transformation?

\_\_\_\_\_ Yes

5. (1 pt) Is the function  $T(x, y) = xy/(x + y)$  a linear transformation?

\_\_\_\_\_ No

homogeneous, but not additive

6. (1 pt) If  $T(\mathbf{x}) = \begin{bmatrix} 4 & -2 \\ 3 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  is a linear transformation for  $\mathbb{R}^2$  to  $\mathbb{R}^3$ , what is  $T\left(\begin{bmatrix} -5 \\ 8 \end{bmatrix}\right)$ ?

$$\begin{bmatrix} -36 \\ -7 \\ -29 \end{bmatrix}$$

7. (1 pt) Is the set of vectors  $\left\{ \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 23 \\ 7 \\ -8 \end{bmatrix} \right\}$  linearly independent or dependent?

In RREF, the augmented matrix for the homogeneous equation gives

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

\_\_\_\_\_ Dependent.