MATH 220 NAME Math 220 - SPRING 08 - FINAL

FINAL EXAM ~~STUDENT NUMBER~~ Answers

MAY 8, 2008 INSTRUCTOR ______

FORM A SECTION NUMBER ______

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your answer sheet. On your answer sheet, identify your name, this course (MATH 220) and the date. Code and blacken the corresponding circles on your answer sheet for your student I.D. number and the class section number. Code in your test form.

There are 25 multiple choice questions each worth 5 points. For each problem, four possible answers are given, only one of which is correct. You should solve the problem, note the letter of the answer that you wish to give and **blacken** the corresponding space on the **answer sheet**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). Check frequently to be sure the problem number on the test sheet is the same as the problem number of the answer sheet.

THE USE OF A CALCULATOR, CELL PHONE, OR ANY OTHER ELECTRONIC DEVICE IS NOT PERMITTED DURING THIS EXAMINATION.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 25 PROBLEMS ON 14 PAGES (INCLUDING THIS ONE).

1- Find the general solution of the system whose **augmented matrix** is

$$\left[\begin{array}{cccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{array}\right]$$

a) $\begin{cases} x_1 = 5 + 7x_2 - 6x_4 \\ x_2 \quad \text{free} \\ x_3 = -3 + 2x_4 \\ x_4 \quad \text{free} \end{cases}$

b) $\begin{cases} x_1 = -4 + 7x_2 - 3x_3 \\ x_2 \quad \text{free} \\ x_3 = -3 + 2x_4 \\ x_4 \quad \text{free} \end{cases}$

c) $\begin{cases} x_1 = 5 - 7x_2 \\ x_2 \quad \text{free} \\ x_3 = -3 \\ x_4 = 0 \end{cases}$

d) The system is inconsistent.

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -1 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$x_1 = 5 + 7x_2 - 6x_4$
x_2 free
$x_3 = -3 + 2x_4$
x_4 free

2- Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$, and $\mathbf{y} = \begin{bmatrix} h \\ 5 \\ 3 \end{bmatrix}$. For which value(s) of h is $\mathbf{y}$ is in the plane generated by $\mathbf{v}_1$ and $\mathbf{v}_2$?

a) $h = -7$

b) $h = 7/2$

c) For all values of h.

d) There is no such value of h.

y is in Span$\{v_1, v_2\}$ if and only if the equation

$y = x_1 v_1 + x_2 v_2$ is consistent

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & 5 \\ -2 & 8 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & 5 \\ 0 & 2 & 3+2h \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & 5 \\ 0 & 0 & 2h-7 \end{bmatrix}$$

$R_3 \to R_3 + 2R_1$ $\qquad$ $R_3 \to R_3 - 2R_2$

Consistent if $2h - 7 = 0$, that is, $h = 7/2$

3- Let $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$, and Then, which of the following is true?

a) The columns of A span $\mathbb{R}^4$.

b) The columns of A span $\mathbb{R}^3$.

c) The matrix equation $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b}$ in $\mathbb{R}^4$.

d) The matrix equation $A\mathbf{x} = \mathbf{0}$ has more than one solutions.

$$\begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \underset{R_4 \to R_4 - 2R_1}{\overset{R_2 \to R_2 + R_1}{\sim}} \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -4 & 2 & -8 \\ 0 & -6 & 3 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & -6 & 3 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 6 & 3 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$Ax = 0$ has a free variable.

4- Describe the solution set of

$$\begin{aligned} x_1 - 2x_2 - 9x_3 + 5x_4 &= 0 \\ x_2 + 2x_3 - 6x_4 &= 0 \end{aligned}$$

in parametric vector form.

a) $\mathbf{x} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 9 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ 6 \\ 0 \\ 1 \end{bmatrix}$

b) $\mathbf{x} = x_2 \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 9 \\ -2 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ 6 \\ 0 \\ 0 \end{bmatrix}$

c) $\mathbf{x} = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 6 \\ 0 \\ 1 \end{bmatrix}$

d) $\mathbf{x} = x_3 \begin{bmatrix} 5 \\ -2 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 6 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 & -9 & 5 & 0 \\ 0 & 1 & 2 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -5 & -7 & 0 \\ 0 & 1 & 2 & -6 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 2R_2$

$x_1 = 5x_3 + 7x_4$

$x_2 = -2x_3 + 6x_4$

$x_3 = x_3$

$x_4 = x_4$

5- How many pivots does a 13×8 matrix A have, if the columns of A are linearly independent?

a) 5
b) 8
c) 13
d) None of the above

6- Which of the following transformations is linear?

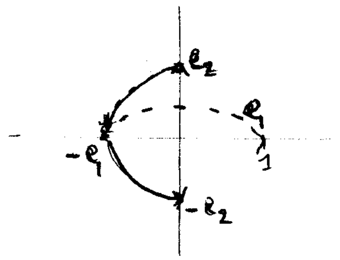
a) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 \\ x_2 - x_3 \\ x_1 \end{bmatrix}$

b) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 x_3 \\ x_2 \\ x_1 + x_2 \end{bmatrix}$

c) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 \\ 1 \end{bmatrix}$

d) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ |x_3| \end{bmatrix}$

7- Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation which reflects points through the vertical x_2-axis and then rotates points $\pi/2$ radians counterclockwise. Find the standard matrix of T.



a) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

b) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

d) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

8- If $A = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$, and $AB = \begin{bmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{bmatrix}$, then what is the first column of B?

a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} -8 \\ -5 \end{bmatrix}$

c) $\begin{bmatrix} 7 \\ 4 \end{bmatrix}$

d) None of the above

Let $B = \begin{bmatrix} a & * & * \\ b & * & * \end{bmatrix}$ Then

$$AB = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} a & * & * \\ b & * & * \end{bmatrix} = \begin{bmatrix} a-2b & * & * \\ -2a+5b & * & * \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{bmatrix}$$

Thus,

$\begin{cases} a - 2b = -1 \\ -2a + 5b = 6 \end{cases}$ which gives $\begin{bmatrix} 1 & -2 & -1 \\ -2 & 5 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 4 \end{bmatrix}$

$a = -1 + 2b = -1 + 8 = 7$

$b = 4$

9- Let $A = \begin{bmatrix} 9 & -14 & 3 \\ -2 & 4 & -1 \\ 3 & -4 & 1 \end{bmatrix}$. Then, which of the following is the third column of A^{-1}?

a) $\begin{bmatrix} 0 \\ -1/2 \\ -2 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 3/2 \\ 4 \end{bmatrix}$

d) A is not invertible.

$AA^{-1} = I$

$$\begin{bmatrix} 9 & -14 & 3 \\ -2 & 4 & -1 \\ 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} * & * & a \\ * & * & b \\ * & * & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} 9a - 14b + 3c = 0 \\ -2a + 4b - c = 0 \\ 3a - 4b + c = 1 \end{cases}$$

$$\begin{bmatrix} 9 & -14 & 3 & 0 \\ -2 & 4 & -1 & 0 \\ 3 & -4 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 3 & -4 & 1 & 1 \\ -2 & 4 & -1 & 0 \\ 9 & -14 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & -4 & 1 & 1 \\ 0 & 4 & -1 & 2 \\ 0 & -2 & 0 & -3 \end{bmatrix} \sim \begin{bmatrix} 3 & -4 & 1 & 1 \\ 0 & 4 & -1 & 2 \\ 0 & 0 & -1 & -4 \end{bmatrix}$$

$R_2 \to 3R_2 + 2R_1$

$R_3 \to R_3 - 3R_1$

$$\boxed{\begin{aligned} a &= 1 \\ b &= 3/2 \\ c &= 4 \end{aligned}}$$

10- Let A be an **invertible** $n \times n$ matrix. Then, which of the following statements is false?

a) The columns of A are linearly independent.

b) $\det A \neq 0$.

c) The linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$ is onto.

d) The equation $A\mathbf{x} = \mathbf{0}$ has a non-trivial solution.

11- Which of the following sets is a subspace of $\mathbb{R}^3$?

a) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1x_2 + x_3 = 0 \right\}$

b) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1^2 - x_2^2 = 0 \right\}$

c) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + 2x_2 - 3x_3 = 0, 4x_2 - x_3 = 0 \right\}$

d) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + x_2 + x_3 = 3 \right\}$

12- Given $A = \begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 5 & 10 & -9 & -7 & 8 \\ 4 & 8 & -9 & -2 & 7 \\ -2 & -4 & 5 & 0 & 6 \end{bmatrix}$, find a basis for the column space of A if A is row equivalent to $\begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \\ 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \\ 0 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 5 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 10 \\ 8 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ -9 \\ -9 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 7 \\ 6 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 5 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ -9 \\ -9 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 7 \\ 6 \end{bmatrix} \right\}$

13- Find the coordinate vector of $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ with respect to the basis $\mathcal{B} = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$,

where $\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$, and $\mathbf{v_3} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

a) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} 3 \\ -3/2 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 \\ 0 \\ -1/2 \end{bmatrix}$

Write

$x = x_1 v_1 + x_2 v_2 + x_3 v_3$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 4 & 2 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1/2 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 1 - 2x_2 - x_3$

$x_2 = -\cancel{1} - \frac{1}{2}x_3 = -3/2$

$x_3 = 1$

$x_1 = 3$

$x_2 = -3/2$

$x_3 = 1$

14- Compute the determinant of $A = \begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ -2 & 6 & 7 & -5 \\ 0 & 0 & 4 & 4 \end{bmatrix}$.

a) 2
b) -2
c) 24
d) -24

$$\det A = -3 \begin{vmatrix} 1 & -2 & 2 \\ -2 & 6 & -5 \\ 0 & 0 & 4 \end{vmatrix} = -3(4) \begin{vmatrix} 1 & -2 \\ -2 & 6 \end{vmatrix}$$

$$= 3(-4)(6-4)$$

$$= -24$$

15- If $\det \begin{bmatrix} a & 5 & 6 \\ b & 1 & 3 \\ c & 2 & 7 \end{bmatrix} = 5$, find $\det \begin{bmatrix} 5 & 1 & 2 \\ 3a & 3b & 3c \\ 6 & 3 & 7 \end{bmatrix}$.

a) 15
b) −15
c) 5/3
d) −5/3

$$\begin{vmatrix} 5 & 1 & 2 \\ 3a & 3b & 3c \\ 6 & 3 & 7 \end{vmatrix} = 3 \begin{vmatrix} 5 & 1 & 2 \\ a & b & c \\ 6 & 3 & 7 \end{vmatrix} = -3 \begin{vmatrix} a & b & c \\ 5 & 1 & 2 \\ 6 & 3 & 7 \end{vmatrix}$$

$$= -3 \begin{vmatrix} a & 5 & 6 \\ b & 1 & 3 \\ c & 2 & 7 \end{vmatrix} = -3(5)$$

16- Which of the following is an eigenvector of $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$?

a) $\begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$A \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 9 \end{bmatrix} = 9 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

17- Find the characteristic equation of $A = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.

a) $(-1-\lambda)(4-\lambda)(2-\lambda)$
b) $4(\lambda^2-\lambda-3)$
c) $(4-\lambda)(\lambda^2-\lambda-3)$
d) $(-1-\lambda)(4-\lambda)(3-\lambda)$

$$\det(A-\lambda I) = \begin{vmatrix} -1-\lambda & 0 & 1 \\ -3 & 4-\lambda & 1 \\ 1 & 0 & 2-\lambda \end{vmatrix} = (4-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix}$$
$$= (4-\lambda)\left((1-\lambda)(2-\lambda)-1\right)$$
$$= (4-\lambda)(\lambda^2-\lambda-3$$

18- Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Which of the following matrices is similar to A?

a) $D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
b) $D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$
c) $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
d) A is not diagonalizable.

$$\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2-1$$
$$= (1-\lambda)(3-\lambda)$$

Eigenvalues of A:
$\lambda = 1$ and $\lambda = 3$

19- Let $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$. Find the dimension of the eigenspace corresponding to the eigenvalue $\lambda = 2$.

a) 1
b) 2
c) 3
d) 4

$$A - 2I = \begin{bmatrix} -1 & 1 & 1 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$(A-2I)X = 0$ has 2 free variables

20- Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the counterclockwise rotation by 90°. Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ be the standard matrix of T. Then which of the following is false?

a) A is similar to the identity matrix.
b) The eigenvalues of A are i and $-i$.
c) $\begin{bmatrix} 1 \\ i \end{bmatrix}$ is an eigenvector of A.
d) A has no eigenvectors on $\mathbb{R}^2$.

We know that similar matrices have the same eigenvalues.

But $\det(A - \lambda I) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1$

The eigenvalues of A are $\lambda_1 = i$ and $\lambda_2 = -i$

the eigenvalues of the identity matrix are $\lambda_1 = 1$ and $\lambda_2 = 1$

21- Which of the following is a unit vector in the same direction as v = $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$?

a) $\begin{bmatrix} 1/9 \\ -2/9 \\ 2/9 \end{bmatrix}$

b) $\begin{bmatrix} -1/9 \\ 2/9 \\ -2/9 \end{bmatrix}$

c) $\begin{bmatrix} -1/3 \\ 2/3 \\ -2/3 \end{bmatrix}$

d) $\begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix}$

$\|v\| = \sqrt{1+(-2)^2+2^2} = \sqrt{9} = 3$

22- Let $\mathbf{v_1} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$, and $\mathbf{v_3} = \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{bmatrix}$. Then, $\mathcal{B}\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is an **orthonormal** basis for $\mathbb{R}^3$. Find the coordinate vector of $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ with respect to $\mathcal{B}$.

a) $\begin{bmatrix} 1/\sqrt{3} \\ 2/\sqrt{2} \\ -2/\sqrt{6} \end{bmatrix}$

b) $\begin{bmatrix} -1/\sqrt{3} \\ 2/\sqrt{2} \\ 2/\sqrt{6} \end{bmatrix}$

c) $\begin{bmatrix} 1/\sqrt{3} \\ -2/\sqrt{2} \\ -2/\sqrt{6} \end{bmatrix}$

d) $\begin{bmatrix} 1/\sqrt{3} \\ 2/\sqrt{2} \\ 2/\sqrt{6} \end{bmatrix}$

$X = x_1 V_1 + x_2 V_2 + x_3 V_3$

$V_1 \cdot V_2 = 0$, $V_1 \cdot V_3 = 0$, $V_2 \cdot V_3 = 0$

$\|V_1\| = 1$, $\|V_2\| = 1$ $\|V_3\| = 1$

$\cancel{x_1}$

$x_1 = x \cdot V_1 = 1/\sqrt{3} - 1/\sqrt{3} + \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$x_2 = x \cdot V_2 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$x_3 = x \cdot V_3 = -2/\sqrt{6}$

$X = \frac{1}{\sqrt{3}} V_1 + \frac{2}{\sqrt{2}} V_2 - 2/\sqrt{6}\, X_3$

$[\cancel{x}]_{\mathcal{B}} = \begin{bmatrix} 1/\sqrt{3} \\ 2/\sqrt{3} \\ -2/\sqrt{6} \end{bmatrix}$

23- Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, and $W = \text{Span}\{\mathbf{v_1}, \mathbf{v_2}\}$. Note that $\{\mathbf{v_1}, \mathbf{v_2}\}$ is an orthogonal basis for W as a subspace of $\mathbb{R}^4$. Write $\mathbf{y} = \begin{bmatrix} 0 \\ 7 \\ 12 \\ 4 \end{bmatrix}$ as the sum of two vectors, one in W, and the other in $W^\perp$, the orthogonal complement of W.

a) $\mathbf{y} = \begin{bmatrix} 0 \\ 7 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 12 \\ 4 \end{bmatrix}$

b) $\mathbf{y} = \begin{bmatrix} 0 \\ 7 \\ 12 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

c) $\mathbf{y} = \begin{bmatrix} 0 \\ 10 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \\ 9 \\ 3 \end{bmatrix}$

d) None of the above.

$y = \hat{y} + z$, where

$$\hat{y} = \frac{0+21+12+0}{1+9+1}\begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix} + \frac{0+17+0+4}{9+1+1}\begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{y} = 3\begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} 0 \\ 10 \\ 3 \\ 1 \end{bmatrix}$$

$$z = y - \hat{y} = \begin{bmatrix} 0 \\ -3 \\ 9 \\ 3 \end{bmatrix}$$

24- Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by the columns of the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

25- Find the distance from the vector $\mathbf{y} = \begin{bmatrix} -1 \\ 5 \\ 6 \\ -5 \end{bmatrix}$ to the line going through the origin and the vector $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix}$.

a) $\sqrt{15}$
b) $2\sqrt{15}$
c) $\sqrt{27}$
d) $\sqrt{87}$

$$\hat{y} = \frac{y \cdot u}{u \cdot u} u = \frac{-2-5-18-5}{4+1+9+1} \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix} = -2 \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} -4 \\ 2 \\ 6 \\ -2 \end{bmatrix} \qquad y - \hat{y} = \begin{bmatrix} 3 \\ 3 \\ 0 \\ -3 \end{bmatrix}$$

$$\|y - \hat{y}\| = \sqrt{9+9+9} = \sqrt{27}$$