MATH 220 NAME ______________________

FINAL EXAMINATION STUDENT NUMBER ______________________

MAY 2, 2006 INSTRUCTOR ______________________

FORM A SECTION NUMBER ______________________

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your answer sheet. On your answer sheet identify your name, this course (Math 220) and the date. Code and blacken the corresponding circles on your answer sheet for your student I.D. number and class section number. Code in your test form. FIVE POINTS WILL BE DEDUCTED FROM YOUR FINAL SCORE IF YOU DO NOT FILL IN YOUR ID NUMBER, SECTION NUMBER OR TEST VERSION CORRECTLY.

There are 25 multiple choice questions each worth six points. For each problem **four** possible answers are given, only one of which is correct. You should solve the problem, note the letter of the answer that you wish to give and **blacken** the corresponding space on the **answer sheet**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). Check frequently to be sure the problem number on the test sheet is the same as the problem number of the answer sheet.

THE USE OF A CALCULATOR, CELL PHONE, OR ANY OTHER ELECTRONIC DEVICE IS NOT PERMITTED DURING THIS EXAMINATION.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 25 PROBLEMS ON 14 PAGES (INCLUDING THIS ONE).

1. Which of the following is the coefficient matrix for a homogeneous system $A\mathbf{x} = 0$ with only the trivial (zero) solution?

a) $A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ 1 free variable

b) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ 1 f.v.

c) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ 1 f.v.

d) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ No f.v.

2. Find the condition that b_1, b_2, b_3 must satisfy in order for the linear system
$x_1 + x_2 + 2x_3 = b_1$
$x_1 + 2x_2 + 3x_3 = b_2$ to be consistent.
$3x_1 + 4x_2 + 7x_3 = b_3$

a) $b_3 = -2b_1 + 5b_2$.

b) $b_3 = 3b_2$.

c) $b_3 = 2b_1 + b_2$.

d) The system is consistent for any b_1, b_2, b_3.

$$[A\ b] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 \\ 1 & 2 & 3 & b_2 \\ 3 & 4 & 7 & b_3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 \\ 0 & 1 & 1 & b_2-b_1 \\ 0 & 1 & 1 & b_3-3b_1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 \\ 0 & 1 & 1 & b_2-b_1 \\ 0 & 0 & 0 & \underbrace{b_3-b_2-2b_1}_{=0} \end{array}\right]$$

3. Suppose $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly independent set of vectors in $\mathbb{R}^3$. Which of the following sets is also linearly independent?

a) $\{\mathbf{u}, \mathbf{u}-\mathbf{v}, \mathbf{u}-\mathbf{w}\}$

b) $\{\mathbf{0}, \mathbf{v}\}$ X

c) $\{\mathbf{u}, \mathbf{u}+\mathbf{v}, \mathbf{u}+\mathbf{w}, \mathbf{v}-\mathbf{w}\}$ X $0 \cdot u + (-1)\cdot(u+v) + 1\cdot(u+w) + 1\cdot(v-w) = 0$.

d) $\{\mathbf{u}, \mathbf{u}-\mathbf{v}, \mathbf{v}\}$ X $(-1)\cdot u + 1\cdot(u-v) + 1\cdot v = 0$.

4. Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 0 \\ 0 & 2 & -2 \end{bmatrix}$, then the span of the columns of A is:

a) $\{\mathbf{0}\}$.

b) a line.

c) a plane.

d) all of $\mathbb{R}^3$.

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 0 \\ 0 & 2 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & -6 \\ 0 & 2 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & -6 \\ 0 & 0 & -14 \end{bmatrix}$$

3 basic variables

5. Suppose $S : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is given by $S(x_1, x_2) = (0,0)$ and $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is given by $T(x_1, x_2, x_3) = (x_1, 2x_2, x_3 + 3)$, then which of the following is true ?

a) Neither S nor T is a linear map. X S is linear. T is not.

b) S is a linear map but T is not.

c) S is not a linear map, but T is.

d) Both S and T and linear maps.

6. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear map. Suppose that $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. What is $T\left(\begin{bmatrix} 3 \\ -4 \end{bmatrix}\right)$?

a) $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$

b) $\begin{bmatrix} 8 \\ 7 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 17 \\ 8 \end{bmatrix}$

$$c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

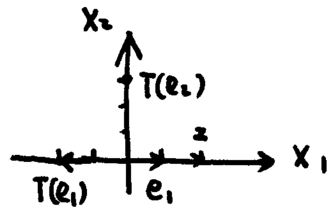
Solve it : $c_1 = 2$, $c_2 = -1$.

$$T\left(\begin{bmatrix} 3 \\ -4 \end{bmatrix}\right) = T\left(c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$$

$$= c_1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

7. Find the standard matrix of the linear map $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ which first expands in the x_1-direction by a factor of two and in the x_2-direction by a factor of three, then reflects across the x_2-axis.



a) $\begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$

c) $\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

d) $\begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$

8. Suppose $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix}$. Find $A^2 - BC + 2A^T$.

a) $\begin{bmatrix} 5 & 3 \\ 0 & 9 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 \\ 0 & 9 \end{bmatrix}$

c) $\begin{bmatrix} 5 & 6 \\ -6 & 9 \end{bmatrix}$

d) $\begin{bmatrix} 11 & 12 \\ 7 & 3 \end{bmatrix}$

9. Given that $A = \begin{bmatrix} 4 & -2 & 0 \\ 5 & 3 & 2 \\ -1 & 1 & 0 \end{bmatrix}$ is an invertible matrix. What is the second column of A^{-1}?

a) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1/3 \\ -1 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} 1/4 \\ 0 \\ 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix}$

Solve $\begin{array}{c} \quad A \qquad\qquad e_2 \end{array}$

$$\left[\begin{array}{ccc|c} 4 & -2 & 0 & 0 \\ 5 & 3 & 2 & 1 \\ -1 & 1 & 0 & 0 \end{array}\right]$$

10. Let A be an $n \times n$ matrix and suppose the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then which of the following statements is true?

a) $\mathbf{b}$ is in the column space of A. X

b) A is row equivlant to I_n. X

c) A has less than n pivot columns.

d) A^T is invertible. X

11. Which of the following is a subspace of $\mathbb{R}^3$?

a) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 < x_2 \right\}$. X

b) The x_1, x_2 - plane.

c) The point $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$. X

d) The set of vectors of the form $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ for any scalar t. X $\left\{ \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} : t \in \mathbb{R} \right\}$ does not include $\vec{0}$.

12. Let A be an $m \times n$ matrix. Which of the following is always true?

a) Rank $A = m$. X

b) The column space of A is a subspace of $\mathbb{R}^n$. X $\mathbb{R}^m$

c) The null space of A has dimension n. X

d) Rank A+dim Nul $A = n$

15. Find the least squares solution $\hat{\mathbf{x}}$ of $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 0 \\ 1 & 3 \\ 1 & 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$.

a) $\hat{\mathbf{x}} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Solve $A^T A x = A^T b$.

b) $\hat{\mathbf{x}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

c) $\hat{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

d) $\hat{\mathbf{x}} = \begin{bmatrix} 4/3 \\ 2/3 \end{bmatrix}$

16. Let A be an $n \times n$ matrix. Which of the following is NOT always true?

a) The characteristic equation of A is of the form $p(\lambda) = 0$, where p is a degree n polynomial. T

b) If A is similar to B then A and B have the same eigenvalues. T

c) A has at most n distinct eigenvalues. T

d) If 1 is an eigenvalue of A, then A is invertible.

17. Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, noting that $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$. What is A^3?

a) $\begin{bmatrix} 5 & 17 \\ -4 & 22 \end{bmatrix}$

b) $\begin{bmatrix} 32 & 0 \\ 15 & 2 \end{bmatrix}$

c) $\begin{bmatrix} -13 & 42 \\ -7 & 22 \end{bmatrix}$

d) $\begin{bmatrix} 16 & -32 \\ -8 & 16 \end{bmatrix}$

$$A = PDP^{-1}$$
$$A^3 = PD^3P^{-1}$$
$$= \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1^3 & 0 \\ 0 & 2^3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

18. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis for $\mathbb{R}^3$, and let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear map such that $[T]_{\mathcal{B}} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$. What is $T(\mathbf{b}_1 + \mathbf{b}_2 - \mathbf{b}_3)$?

a) $3\mathbf{b}_3$

b) $\mathbf{b}_1 + \mathbf{b}_2 - \mathbf{b}_3$

c) $3\mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$

d) $2\mathbf{b}_1$

$$T(b_1 + b_2 - b_3)$$
$$= T(b_1) + T(b_2) - T(b_3).$$
$$= (2b_1 + b_2) + (b_1 - b_2 + b_3) - (3b_1 - 2b_3)$$
$$= 3b_3$$

19. Let A be a 3×3 matrix with real number entries and suppose that $\lambda_1 = 3$ and $\lambda_2 = 1 + 2i$ are two of its eigenvalues. What is the third (real or complex) eigenvalue of A?

a) It cannot be determined from the information given

b) -3

c) $-1 + 2i$

d) $1 - 2i$

$\overline{\lambda_2}$

20. What is the distance between $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$?

a) $\sqrt{11}$

b) $\sqrt{5}$

c) 9

d) 5

$|u - v| = \sqrt{0^2 + (-3)^2 + (-1)^2 + (-1)^2}$

21. Which of the following statements is NOT always true?

a) If U is an orthogonal matrix, then U is invertible. T

b) If U is an orthogonal matrix, then $\det U > 0$. F $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is orthogonal.

c) The columns of an orthogonal matrix form an orthonormal set. T

d) If U is an $n \times n$ orthogonal matrix, then $\|U\mathbf{x}\| = \|\mathbf{x}\|$ for all vectors $\mathbf{x}$ in $\mathbb{R}^n$. T

22. Given that $\mathcal{B} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal set, where $\mathbf{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$, what is $[\mathbf{x}]_{\mathcal{B}}$ for $\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$?

Solve
$$\begin{array}{cccc} u_1 & u_2 & u_3 & x \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 2 \\ -3 & 2 & 1 & 4 \\ 0 & -1 & 4 & 6 \end{array}\right]$$

a) $\begin{bmatrix} -1/3 \\ 2/3 \\ 5/3 \end{bmatrix}$

b) $\begin{bmatrix} 2/3 \\ 4/3 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} -6 \\ 6 \\ 30 \end{bmatrix}$

d) None of the above

23. Let W be the subspace of $\mathbb{R}^3$ spanned by $\begin{bmatrix}1\\3\\5\end{bmatrix}$ and $\begin{bmatrix}1\\3\\-2\end{bmatrix}$, and let $\mathbf{y} = \begin{bmatrix}1\\1\\1\end{bmatrix}$. What is the orthogonal projection of $\mathbf{y}$ onto W?

$u_1 \quad u_2$

a) $\begin{bmatrix}11\\33\\41\end{bmatrix}$

b) $\begin{bmatrix}-1\\-5\\-2\end{bmatrix}$

c) $\begin{bmatrix}2/5\\6/5\\1\end{bmatrix}$

d) $\begin{bmatrix}3/5\\-1/5\\0\end{bmatrix}$

$$\hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + \frac{y \cdot u_2}{u_2 \cdot u_2} u_2$$

$$= \frac{9}{35}\begin{bmatrix}1\\3\\5\end{bmatrix} + \frac{2}{14}\begin{bmatrix}1\\3\\-2\end{bmatrix}$$

24. Use the Gram-Schmidt process to determine an orthogonal basis for the subspace W of $\mathbb{R}^3$ spanned by $\mathbf{u}_1 = \begin{bmatrix}1\\0\\-1\end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix}2\\1\\4\end{bmatrix}$.

a) $\left\{\begin{bmatrix}2\\1\\4\end{bmatrix}, \begin{bmatrix}2\\0\\-1\end{bmatrix}\right\}$

b) $\left\{\begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}3\\1\\3\end{bmatrix}\right\}$

c) $\left\{\begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}2\\0\\2\end{bmatrix}\right\}$

d) $\left\{\begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$

$$v_1 = u_1$$

$$v_2 = u_2 - \frac{u_2 \cdot v_1}{v_1 \cdot v_1} v_1$$

$$= \begin{bmatrix}2\\1\\4\end{bmatrix} - \frac{-2}{2}\begin{bmatrix}1\\0\\-1\end{bmatrix}$$

$$= \begin{bmatrix}2\\1\\4\end{bmatrix} + \begin{bmatrix}1\\0\\-1\end{bmatrix}$$

13. Let $A = \begin{bmatrix} 5 & 2 & 0 \\ -1 & 3 & 1 \\ 4 & -2 & 0 \end{bmatrix}$, then what is det A?

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

a) -18

b) 18

c) 12

d) -12

$$\det A = -1 \det\left(\begin{bmatrix} 5 & 2 \\ 4 & -2 \end{bmatrix}\right)$$

$$= -1(-10-8)$$

14. Let $A = \begin{bmatrix} -4 & 0 & 5 \\ 1 & 2 & 1 \\ 7 & 6 & -3 \end{bmatrix}$, and $B = \begin{bmatrix} 7 & 6 & -3 \\ 1 & 2 & 1 \\ -4 & 0 & 5 \end{bmatrix}$. Which of the following is true?

a) det $A = 0$.

b) det $B = 2$.

c) det $A =$det B.

d) det $A = -$det B.

$A \longrightarrow B$ change order of row 1 and row 3.

25. Which of the following statements is NOT always true?

A is sym. $A = PDP^T$ $A^{-1} = PD^{-1}P^T$

a) If A is an invertible symmetric matrix, then A^{-1} is orthogonally diagonalizable. T

b) If A is similar to a symmetric matrix B, then A is orthogonally diagonalizable.

c) If A is orthogonally diagonalizable, then A is symmetric. T

d) If A is orthogonally diagonalizable, then so is A^2. T