# Brief survey of research papers by S. Tabachnikov

## Homology of infinite-dimensional Lie algebras. Invariant differential operators [1, 3, 7, 24]

The algebras in question are the Lie algebras of formal vector fields in $\mathbf{R}^n$; the homology are with coefficients in tensor products of moduli of various tensor fields multiplied by $\lambda$-densities. This homology depends on $\lambda$, and for $\lambda$ in general position the homology has minimal dimension possible. I constructed a spectral sequence for computation of homology in general position. It was applied to the classification problem of invariant differential operators posed by O. Veblen in the 1920s. The obtained estimates for the dimension of the space of invariant differential operators in general position on the line remain the best known.

In [24] the Vey 2-cocycle of the Poisson Lie algebra of a symplectic manifold ("responsible" for deformation quantization) is integrated to a group cocycle using projective structures. This construction is closely related with deformation quantization.

## Characteristic classes of foliations [2, 4, 5, 6, 7, 10]

A homogeneous foliation is a foliation of the quotient of a semi-simple Lie group $G$ by its cocompact discrete subgroup, whose leaves are the conjugate classes of a Lie subgroup $H \subset G$. The object of study were characteristic classes of homogeneous foliations with $G$ a classical group of series $A, B, C, D$ and $H$ its parabolic subgroup. A complete description of these classes was obtained. As a byproduct a number of new combinatorial identities with symmetric polynomials was found.

In [10], characteristic classes of Lagrangian and Legendrian foliations on symplectic and contact manifolds were constructed and computed in particular cases.

## Contact topology and Legendrian knot theory [8, 9, 22, 26, 30]

Transverse and Legendrian knots in contact space have self-linking numbers, called the Bennequin and the Thurston-Bennequin invariants, respectively. In [8] self-linking is generalized to the case of submanifolds transverse to a codimension-one distribution; this invariant is computed for the link of an isolated holomorphic singularity in $\mathbf{C}^n$ in terms of its Milnor number.

In [9] the Thurston-Bennequin invariant is computed in terms of the geometry of its front, leading to Plucker-like global formulas for immersed curves. This work was later extended by a number of authors, in particular, E. Ferrand.

[26] concerns transverse and Legendrian knots in the standard contact space. One of the main results is that contact invariants of finite type cannot distinguish between topologically isotopic Legendrian knots with equal Maslov and Thurston-Bennequin numbers; a similar result holds for transverse knots too. Another result is an estimate above for the Thurston-Bennequin number of Legendrian and transverse knots and links within a topological isotopy class; the answer is given in terms of the Homfly and Kauffman knot polynomials. In [30] the above estimates are deduced from state models for knot polynomials. By now, many similar inequalities for self-linking are known in the fast growing Legendrian knot theory, in particular, in the work of L. Ng.

[22] concerns triple point free immersed plane curves. New invariants of such curves are constructed that take value in the free group with 2 generators. A number of conjectures on the value of V. Arnold's $St$-invariant of plane curves are proved.

**Around the four vertex theorem** [12, 20, 23, 27, 28, 38, 39, 60]

In [23] a Sturm-theoretical and contact geometry proof is given of Ghys' theorem that the Schwarzian derivative of a projective line diffeomorphism vanishes in at least 4 distinct points. In [27] a stronger result is proved: the number of zeroes of a projective line diffeomorphism is not less than the number of its fixed points. These results are related to V. Arnold's conjectures on the number of vertices, some of which were later proved by Chekanov and Pushkar using methods of Legendrian knot theory.

In [39] discrete versions of the 4-vertex, the 6 affine vertex and the Ghys theorems are given; this work is closely related to [38] which provides a family of 4-vertex type theorems for plane polygons.

In [20] the following observation is made: the envelope of the lines generated by the acceleration vectors of a parameterized convex closed plane curve has at least 4 cusps. This is explained in [28] from the view-point of contact geometry and Sturm theory.

In [60] I prove that if $V^n$ is a Chebyshev system on the circle and $f$ is a function with at least $n+1$ sign changes then there exists an orientation preserving diffeomorphism of $S^1$ that takes $f$ to a function $L^2$-orthogonal to $V$. This is a converse Sturm-Hurwitz-Kellogg theorem.

**Inner and outer billiards, and billiard-like systems** [13, 16, 17, 19, 21, 25, 31, 35, 40, 44, 45, 52, 53, 56, 59, 61, 75]

The outer billiard transformation acts in the exterior of a convex plane domain reflecting a point $x$ in the tangency point of the support line through $x$ to the domain. The study of outer billiards was initiated by J. Moser in the 1970s.

In [16, 19] the area spectrum of the outer billiard transformation is introduced, its asymptotic expansion near the dual billiard curve is obtained, and it is proved that ellipses are characterized by their area spectrum. The construction of outer billiards is extended to the case of a convex hypersurface in linear symplectic space; it is proved that the outer billiard transformation is a symplectomorphism and that it has periodic orbits of all odd prime periods. Outer billiard about a regular pentagon is analyzed: a symbolic dynamical description of the map is given and the Hausdorff dimension of the fractal set of infinite orbits is computed.

In [13, 21] the asymptotic dynamics of outer billiards at infinity is shown to be Hamiltonian and to satisfy a kind of Kepler's Law. A conjecture made in [13] about orbits escaping to infinity was recently proved by Dolgopyat and Fayad. In [17] it is proved that if two outer billiard maps commute then the respective billiard tables are homothetic ellipses. [44, 45] is the first study of outer billiards, smooth and polygonal, in the hyperbolic plane. One of the main results of [52] is that the complexity function of plane polygonal outer billiards has a polynomial growth (this is an open problem for inner billiards). [53] is an application of sub-Riemannian geometry to outer billiard with 1-dimensional sets of periodic points.

[56] contains a proof that polygonal outer billiards always possess periodic orbits (also an open problem for inner billiards). [59] is a proof of an algebraic version of Birkhoff's conjecture for outer billiards: the only algebraically integrable plane outer billiard is an ellipse. In [25, 31] a new class of billiard-like systems called projective billiards is introduced and studied. [40] is a study of billiards in Finsler and Minkowski geometries. [61] is a proof that Birkhoff billiards are insecure (a manifold is secure if, for any pair of points $x, y$ there exists a finite set $S$ such that every geodesic from $x$ to $y$ contains a point in $S$).

In [75] we give a detailed description of the symbolic periodic trajectories on the double regular pentagon, including their periods. This extends a recent work by Smillie and Ulcigrai on the regular octagon, another interesting Veech surface.

**Affine and projective differential geometry** [14, 15, 33, 36, 49, 54, 55, 63, 74]

A Lagrangian 2-web is a pair of transverse Lagrangian foliations in a symplectic manifold; a Legendrian 2-web is a pair of Legendrian foliations in a contact manifold generating the contact distribution at each point. In [14] local invariants of these structures are constructed, related to the classical results of E. Cartan, S.-S. Chern and J. Moser.

In [15] the Poncelet theorem is deduced from complete integrability of outer billiard system in the hyperbolic plane, and in [54] the symmetries of the Poncelet grid are explained via billiard inside an ellipse. This work is also related to classical results of Darboux.

[49] contains a solution to an analog of Hilbert's 4th problem for Finsler metrics whose geodesics are circles. The Carathéodory conjecture states that a convex surface in $\mathbf{R}^3$ has at least two umbilic points; [55] concerns a projective analog of this conjecture for saddle-like surfaces in $\mathbf{RP}^3$. [63] is a classification of projectively self-dual polygons in $\mathbf{RP}^2$, a partial solution to V. Arnold's problem.

[33, 36] introduce exact transverse line fields along hypersurfaces in affine and projective spaces and study their geometry and topology. This theory is related to symplectic, Finsler and projective geometry. For example, a transverse line field $\xi$ along a sphere $\mathbf{S}^n$ is exact if and only if the $n$-dimensional submanifold of the space of oriented lines in $\mathbf{R}^{n+1}$ determined by $\xi$ is Lagrangian with respect to the symplectic structure, associated with the hyperbolic metric inside the sphere (the Klein model).

[74] contains a discrete version of Schwartz's inequality on the Schwarzin derivative: if all solutions of Hill's equation $f''(t) + k(t)f(t) = 0$ with a $T$-periodic potential $k$ are also $T$-periodic then $T\int_0^T k(t)\ dt \le \pi^2$ with equality only for constant potential $k(t)$ (this is an extension of the classical Blaschke-Santalo inequality to star-shaped curves). Our inequalities concern equiafine geometry of plane polygons; they are also interpreted in terms of Coxeter's frieze patterns.

**Complete integrability** [34, 41, 57, 64, 68]

In [34, 41] a new mechanism of complete integrability was discovered: if a manifold carries two metrics with the same non-parameterized geodesics then the geodesic flow of each metric is completely integrable. This is applied to the geodesic flow on ellipsoids, the second metric being induced by the hyperbolic metric insider the ellipsoid (the Klein model). This mechanism of complete integrability was independently and simultaneously discovered and studied in a series of papers by V. Matveev and P. Topalov.

[57, 64] concerns the geodesic flow on and the billiard inside an ellipsoid in pseudo-Euclidean space and proves their complete integrability. The space of light-like geodesics carries a contact structure, in contrast with the spaces of space- and time-like geodesics that are symplectic. In particular, in [57] we prove a Poncelet-style theorem for null geodesics on an ellipsoid in Minkowski space. This work was recently extended by V. Dragovic and M. Radnovic.

In [68] we develop the theory of completely integrable systems on contact manifold, akin to the classical Arnold-Liouville integrability.

**Applications of algebraic topology** [42, 43, 46, 48, 72]

[42, 43] study the $D_n$-equivariant cohomology ring of the cyclic configuration space of the sphere. An application is a multi-dimensional version of classical Birkhoff's theorem that the billiard in a smooth strictly convex plane oval has at least two periodic trajectories for every period and every rotation number. We prove the multi-dimensional theorem using equivariant Morse theory. This work was continued in papers of F. Duzhin and R. Karasev. [48] provides a lower bound on the number of 3-periodic trajectories of multi-dimensional outer billiard.

[46] concerns the complexity of motion planning problem in $\mathbf{RP}^n$. The main result is that this problem is equivalent to the immersion problem for projective spaces. The literature on the complexity of motion planning problem is growing fast (the state of the art is described in M. Farber's forthcoming monograph).

[72] concerns topological aspects of the celebrated Dvoretzky theorem that every convex body has almost round low-dimensional sections: a number of results of this kind are proved for general families of convex bodies continuously assigned to every $k$-dimensional subspace in $n$-dimensional space. This work was recently extended by R. Karasev.

**Non-degenerate immersions and embeddings** [47, 51, 58, 67]

A skew loop is a closed curve in $\mathbf{R}^3$ whose tangent lines at distinct points are not parallel; a skew brane $M^n \subset \mathbf{R}^{n+2}$ is a submanifold without parallel pairs of tangent spaces. [47] proves nonexistence of skew branes on a quadratic hypersurface and of skew loops on embedded ruled, developable disks. In [67] it is proved that the Euler characteristic is an obstruction to being a skew brane, and the first multidimensional examples of a skew odd dimensional spheres are constructed.

A totally skew submanifold in an affine or projective space is a manifold such that any pair of tangent lines at two distinct points are not coplanar. Given a manifold $M$, the problem is to find the least dimension $N(M)$ of Euclidean space into which $M$ can be embedded as a totally skew submanifold. [58] gives various lower and upper bounds on $N(M)$ for $M$ a disc and a sphere. The bounds are obtained using methods of algebraic topology. For example, if $N(\mathbf{R}^k) = 2k+1$ (the a priori lowest possible value) then $k \in \{1, 3, 7\}$. This work was recently extended by R. Zivaljevic and his co-authors, and by R. Karasev.

A $T$-embedded manifold is characterized by the property that the tangent spaces at distinct points do not intersect. In [51] we prove that there exist no $T$-embedded $n$-dimensional discs in $\mathbf{R}^{2n}$.

**Nonholonomic dynamics. Tire track geometry** [50, 65]

We model bicycle by a segment of fixed length $L$ allowed to move in the plane in such a way that the velocity of the rear end of the segment is always aligned with the segment (this is justified by the fact that the rear wheel is fixed on the bicycle frame, whereas the front one can steer). The same model describes the motion of a skate (or a knife blade), and this is one of the most popular examples in nonholonomic dynamics where it is often referred to as the Chaplygin skate. Still the same model describes the so-called hatchet, or Prytz, planimeter, a mechanical device that measures areas of plane domains. The configuration space of the bicycle is the space of (co)oriented contact elements in the plane, and the restriction on the motion defines the canonical contact structure in the space of contact elements. This description extends to any Riemannian surface.

Given the trajectories of the front and the rear wheels, typically one can tell which way the bicycle went. However, there are exceptional pairs of trajectories for which the direction of motion is impossible to determine. The description of such ambiguous pairs is an open problem which is, surprisingly, equivalent to the 2-dimensional Ulam's problem from flotation theory: which bodies with uniform density float in equilibrium in all positions? In [50] this problem is studied and a number of results is obtained: examples of non-trivial solutions, a rigidity criterion in linear approximation, rigidity results for small rotation numbers (corresponding to the relative density in the flotation formulation), a discrete polygonal version of the problem. The last problem was later studied by R. Connelly and B. Csikos.

If one knows the trajectory $\gamma$ of the front wheel, then the trajectory of the rear one is determined only if the initial position of the bicycle is specified. A monodromy map arises $T_\gamma : S^1 \to S^1$. In [65] we show that this monodromy is always a Moebius transformation

($S^1 = \mathbf{RP}^1$ via stereographic projection) and give a multi-dimensional version of this result. For a closed curve $\gamma$, one has a trichotomy: $T_\gamma$ may be elliptic, parabolic or hyperbolic (having no, one, or two fixed points). We proved the 100 years old Menzin's conjecture (originally concerning hatchet planimeters): if $\gamma$ is a closed convex curve bounding area greater than $\pi L$ then the respective monodromy is hyperbolic.

**Pentagram map and related topics** [62, 66, 69, 70, 73, 77, 79, 80]

The Pentagram map is a geometrically natural iteration on projective equivalence classes of polygons in the projective plane: the new polygon is spanned by the intersection points of consecutive "short" diagonals of the initial one. The Pentagram map was introduced and studied by R. Schwartz. The Pentagram map $T$ is a completely integrable system: it was shown in [62, 69] that the larger moduli space of equivalence classes of polygons with monodromy (twisted polygons) in the projective plane has a $T$-invariant Poisson structure and the right number of independent integrals in involution for the Arnold-Liouville integrability to hold. In [79] we extend the main result of [62, 69] on complete integrability of the pentagram map to closed polygons. In a different way, this was also proved by F. Soloviev in a recent paper.

It was observed in computer experiments that, for polygons inscribed into conics, a certain duality between the integrals of the Pentagram map held. This was proved in [73] by combinatorial methods. An interesting byproduct of our study of inscribed polygons was a discovery of 8 new configuration theorems of projective geometry somewhat akin to the classical theorems of Pappus, Pascal and Brianςon [70].

[77] introduces and studies 2-frieze patterns that generalize frieze patterns of Coxeter and Conway. The space of frieze patterns is isomorphic to the moduli space of polygons on which the Pentagram map. This space carries a structure of cluster variety. The cluster algebras aspects of the Pentagram map were recently studied by M. Glick.

In [80] we generalize the pentagram map by including it into a family of completely integrable discrete dynamical systems related with cluster dynamics; this extends the work of M. Glick. We interpret these maps as higher diagonal maps and, in dimension one, as a circle pattern transformation previously studied by O. Schramm.

**Miscellanea** [11, 18, 29, 32, 37, 71, 76, 78]

In [11] the Godbillon-Vey class of codimension one foliations is interpreted dynamically as the asymptotic linking number. This has applications in topological hydrodynamics.

In [18] an explicit construction is given of a homotopy of the vector field grad $(x^2 + y^2)$ to its negative in the class of nondegenerate gradient vector fields in the punctured plane. Simpler than the famous sphere eversion, this construction has a similar $h$-principle flavor.

Here is a sample result from [29]: let $f$ be a trigonometric polynomial of degree $N$, free from the harmonics whose orders are divisible by $n$. Then the relative measure of the set $\{x \,|f(x) < 0\}$ is at least $1/n$, and in the case of equality the number of connected components of this set is at most $N/(n-1)$.

[32] concerns differential geometry of developable surfaces in 3-space: we study paper folding along curves (and not just straight lines). Answering a question of M. Kontsevich, we describe the curves in space which can be obtained by folding a sheet of paper along a prescribed plane curve.

Given a polygon $A_1, ..., A_n$, consider the chain of circles: $S_1$ inscribed in the angle $A_1$, $S_2$ inscribed in the angle $A_2$ and tangent to $S_1$, $S_3$ inscribed in the angle $A_3$ and tangent to $S_2$, etc. For $n = 3$ this process is known to be 6-periodic. In [37] a large class of $n$-gons is described for which this process is $2n$-periodic; this result is extended to the case when the sides of a polygon are circular arcs (a generalization of Money-Coutts theorem).

The Tait-Kneser theorem asserts that the osculating circles of an arc with a monotone curvature are pairwise nested. In [71] we give a series of similar theorems, for example, for osculating Taylor polynomials of even degree and for osculating cubic curves. Similar results were independently discovered by E. Ghys.

[76] is related to spherical design and concerns discrete spherical means of directional derivatives of a function, with applications to nonlinear ltering of digital images.

[78] contains a counter-example to a long-standing conjecture made in [12]: given a convex closed curve $\gamma$ in the plane and a loop $\Gamma$ around it, there exists a point on $\Gamma$ from which the two tangent segments to $\gamma$ have equal lengths. This problem is related with the study of symmetry sets of plane domains and with computer vision.

**Books and book chapters** [81, 82, 85, 84, 85, 86, 87]

The books [81, 85] concern mathematical billiards and related topics; the second book is oriented toward advanced undergraduate and graduate students. [85] is translated into Russian; a German translation is in preparation. The survey [83] concerns polygonal billiards and the related theory of translation surfaces, and [87] surveys the relation between the Poncelet theorem and billiards.

The book [84] is a modern treatment of projective differential geometry and an exposition of the links of this theory with many modern areas of research, such as Virasoro algebra, cohomology of the group of diffeomorphisms of the circle, moduli spaces, etc. It is translated into Russian.

The expository book [86] is for a wide audience, from high school students to accomplished researchers. It consists of thirty lectures on “classic” topics from algebra, arithmetic, geometry, and topology, with the emphasis on the interplay of ideas from different fields and, above all, the beauty and elegance of the subject. The book is translated into Russian and German, a translation into Japanese is in preparation.

[82] was written as a textbook for Gelfand School by Correspondence in Russia, and it is still in use at this School.

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