

Math 251 (§10) Homework 9

Due: Wednesday, April-22-2008

Answers to the following should be turned in no later than the end of class on the above date. Write your name on the assignment. This assignment is worth a total of 50 points.

QUESTION 1: Compute the Fourier series on the appropriate intervals for the following functions. Sketch the graph of f and what its Fourier series converges to on $(-l, l)$.

$$\text{i) } f(x) = \begin{cases} 1 & -2 \leq x < -1 \\ 0 & -1 \leq x < 1 \\ 1 & 1 \leq x \leq 2 \end{cases}$$

$$\text{ii) } f(x) = \begin{cases} 1 & -2 \leq x < -1 \\ x & -1 \leq x < 1 \\ 1 & 1 \leq x \leq 2 \end{cases}$$

QUESTION 2: Sketch the odd extension of the following functions and find its Fourier series on the appropriate intervals.

$$\text{i) } f(x) = \begin{cases} 1 - x & 0 \leq x < 1 \\ 0 & 1 \leq x \leq 2 \end{cases}$$

$$\text{ii) } f(x) = \begin{cases} 0 & 0 \leq x < 2 \\ 2 & 2 \leq x \leq 3 \end{cases}$$

QUESTION 3: Sketch the even extension of the following functions and find its Fourier series on the appropriate intervals.

$$\text{i) } f(x) = \begin{cases} 1 & 0 \leq x < 2 \\ 3 - x & 2 \leq x \leq 3 \end{cases}$$

$$\text{ii) } f(x) = 1 - x \quad 0 \leq x \leq 2$$

QUESTION 4: Solve the following heat equation problem. Show all steps, including separation of variables and calculation of positive eigenvalues/eigenfunctions. You may assume the Euler-Fourier formulas; do not rederive them.

$$u_t = 9u_{xx} \quad u_x(0, t) = u_x(1, t) = 0 \quad u(x, 0) = x + 1$$