

# Math 251 (§3) Homework 8

Due: Thursday, April-17-2008

Answers to the following should be turned in no later than the end of class on the above date. *Write your name on the assignment.* This assignment is worth a total of 50 points.

QUESTION 1: Find all eigenvalues and eigenfunctions for the following boundary condition problems.

i)  $y'' + \lambda y = 0 \quad y(0) = 0 \quad y'(l) = 0$

ii)  $y'' + \lambda y = 0 \quad y(-\pi) = y(\pi) \quad y'(-\pi) = y'(\pi)$

*Hint:* The positive eigenvalues should have two eigenfunctions associated to each.

QUESTION 2: Reduce the following PDEs to a pair of ODEs using separation of variables.

i)  $xu_{xx} + u_t = 0$

ii)  $x^{-1}u_t = (xu_x)_x$

iii)  $u_{xx} + u_{tt} + xu = 0$

QUESTION 3: Find the solution to the following heat conduction problem.

$$u_t = 100u_{xx}$$

$$u(0, t) = u(1, t) = 0$$

$$u(x, 0) = \sin(2\pi x) - \sin(5\pi x)$$

*Hint:* Your solution should be a finite sum of basic solutions  $u_n$ .

QUESTION 4: Suppose we have the heat equation  $u_t = ku_{xx}$  with the following boundary conditions:

$$u(0, t) = 0$$

$$u_x(l, t) + u(l, t) = 0$$

We know that by separating variables, we obtain the pair of ODEs

$$X'' + \lambda X = 0$$

$$T' + \lambda kT = 0.$$

Determine the boundary conditions for  $X$ . For which values of  $\lambda$  do we have nontrivial solutions? You don't have to actually find the positive eigenvalues. Finding a condition that they have to satisfy is sufficient. But do show that there are only positive eigenvalues in addition to this.