

Math 251 (§3) Homework 7

Due: Thursday, April-3-2008

Answers to the following should be turned in no later than the end of class on the above date. *Write your name on the assignment.* This assignment is worth a total of 50 points.

QUESTION 1: Find the general solution, and then the particular solution satisfying the given initial conditions, of the following linear systems.

$$\text{i) } \mathbf{x}' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \mathbf{x} \quad \mathbf{x}(0) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$\text{ii) } \mathbf{x}' = \begin{pmatrix} 2 & -2 \\ 8 & 2 \end{pmatrix} \mathbf{x} \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\text{iii) } \mathbf{x}' = \begin{pmatrix} 4 & 2 \\ -4 & -5 \end{pmatrix} \mathbf{x} \quad \mathbf{x}(0) = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$\text{iv) } \mathbf{x}' = \begin{pmatrix} 0 & 2 \\ -5 & 2 \end{pmatrix} \mathbf{x} \quad \mathbf{x}(0) = \begin{pmatrix} 4 \\ 13 \end{pmatrix}$$

QUESTION 2: For each of the parts of the previous question, sketch the phase plane for the system of differential equations. Also determine the long-term behavior of the particular solutions satisfying the given initial conditions.

QUESTION 3: Let $\mathbf{x}(t)$ be a solution of

$$\mathbf{x}' = \begin{pmatrix} -2 & -6 \\ 1 & 5 \end{pmatrix} \mathbf{x} \quad \mathbf{x}(0) = \begin{pmatrix} a \\ b \end{pmatrix}.$$

For what values of a and b will $\lim_{t \rightarrow \infty} \mathbf{x}(t) = \mathbf{0}$?

QUESTION 4: Find the linearized matrix of the following system, determine the critical points, and sketch the solution space.

$$x' = x - y$$

$$y' = x^2 - 4$$