

Math 251 (§3) Homework 2

Due: Wednesday, 30-Jan-07

Answers to the following should be turned in no later than the end of class on the above date. *Write your name on your assignment.* This assignment is worth 50 points.

QUESTION 1: You are sitting in a hot tub in the snow. Snow enters the tub at a rate of 1 liter per minute and water flows out of the tub at the same rate. Initially, the tub holds 500 liters of water and has a chlorine concentration of 10 milligrams per liter. The filter adds 2 milligrams per minute of chlorine. If we assume the hot tub is well mixed:

- i) Write down and solve the initial value problem for the amount of chlorine in the hot tub.
- ii) If the filter can only add 60 milligrams of chlorine, how much chlorine will be present in the tub when it runs out?

QUESTION 2: A 100 liter well-mixed tank initially contains 100 liters of water contaminated by 20 grams of blue food coloring. Suppose that food coloring is added at a constant rate of a grams per minute while water is drained out at a rate of 1 liter per minute. What value of a should be chosen so that the concentration of coloring is 3 grams per liter when 20 liters remains?

QUESTION 3: For the following DEs, find and classify their critical points and sketch the phase space. Then note the behavior of the solution with the given initial value.

- i) $y' = y^3 - 2y^2 - 5y + 6$, $y(0) = 1$
- ii) $y' = (y + 1) \sin(y)$, $y(0) = \pi$.

QUESTION 4: For each of the following differential equations, state whether they are linear or nonlinear. Then, depending on your answer, conclude what you can from the Fundamental Theorem of Existence and Uniqueness.

- i) $y^3 y' = e^t \tan(yt)$ $y(1) = 17\pi$
- ii) $y' + \frac{1}{x^2-1} = \ln(\sin x)$ $y\left(\frac{1}{10}\right) = 4$
- iii) $\sin(2x)y'' + \frac{1}{2x-1}y' + x^3y = 2$ $y\left(\frac{1}{4}\right) = y'\left(\frac{1}{4}\right) = 3$