

# Math 558: Foundations of Mathematics

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This course is suitable for all mathematics graduate students. The textbook will consist of notes provided by the instructor, also on-line at

<http://www.math.psu.edu/simpson/courses/math558/>

## 1. Computability

Primitive recursive functions, the Ackermann function, computable functions, partial recursive functions, the enumeration theorem, the Halting Problem, examples of functions and sets which are not computable.

## 2. Undecidability of the Natural Number System

Terms, formulas, sentences, arithmetical definability, Chinese Remainder Theorem, definability of computable functions, definability of the Halting Problem, Gödel numbers, undefinability of arithmetical truth.

## 3. Decidability of the Real Number System

Effective functions, quantifier elimination (P. J. Cohen's method), definability over the real number system, decidability of the real number system, decidability of Euclidean geometry.

## 4. Introduction to Set Theory

Russell paradox, operations on sets, cardinal numbers, ordinal numbers, transfinite recursion, the Axiom of Choice, the Well Ordering Theorem, the Continuum Hypothesis, inaccessible cardinals.

## 5. Independence of the Continuum Hypothesis

The Zermelo-Fraenkel axioms, set-theoretic foundations of mathematics, models of set theory, inner models, constructible sets, the inner model  $L$ , the Generalized Continuum Hypothesis in  $L$ , models constructed by forcing, a model where the Continuum Hypothesis fails.