# Math 558 – Midterm Exam #1

February 12, 2007

1. Prove that the function

$$f(n) = \text{the } n\text{th digit of } \sqrt{5}$$

   is primitive recursive.

2. Write a register machine program which computes the function

$$f(n) = 2^n \,.$$

3. Recall that an *index* of a computable function is defined to be the Gödel number of a program which computes the function.

   Find an index of the function

$$\alpha(x) = \begin{cases} 1 & \text{if } x = 0, \\ 0 & \text{if } x > 0. \end{cases}$$

4. (a) Define what it means for a set $C \subseteq \mathbb{N}$ to be $\Pi^0_n$ complete.

   (b) Prove that the set $T = \{x \mid \varphi^{(1)}_x \text{ is total}\}$ is $\Pi^0_2$ complete.

5. Prove the existence of a primitive recursive function $f(x)$ with the following property: for all $x$, if $\varphi^{(1)}_x$ is a permutation of $\mathbb{N}$, then $\varphi^{(1)}_{f(x)}$ is the inverse permutation of $\mathbb{N}$.

   Extra Credit:

   What happens if $\varphi^{(1)}_x$ is assumed only to be partial and one-to-one, and not necessarily a permutation of $\mathbb{N}$?