

Math 558 – Homework #6

April 20, 2007

1. Using abbreviations introduced in the lecture notes, write formal sentences in the language \mathcal{L}_ϵ expressing the following assertions:
 - (a) Cantor's Theorem: $\forall \kappa 2^\kappa > \kappa$.
 - (b) König's Theorem.
 - (c) The existence of the unique smallest set w such that $\{\} \in w \wedge \forall x (x \in w \Rightarrow x \cup \{x\} \in w)$. We introduce the abbreviation ω to denote this set.
 - (d) The Continuum Hypothesis, CH: $2^{\aleph_0} = \aleph_1$.
 - (e) The negation of CH: $2^{\aleph_0} > \aleph_1$.
 - (f) The Generalized Continuum Hypothesis.
2. Prove the Generalized Löwenheim/Skolem Theorem.
3. Prove that R_ω is a transitive model of all of the ZFC axioms except the Axiom of Infinity.
4. Prove that $R_{\omega+\omega}$ is a transitive model of all of the ZFC axioms except (some instances of) the Replacement Scheme. Thus $R_{\omega+\omega}$ is a model of ZC, Zermelo Set Theory with the Axiom of Choice.
5. Let λ be an inaccessible cardinal. We know that R_λ is a model of ZFC. Prove that there exist arbitrarily large cardinals $\kappa < \lambda$ such that R_κ is a model of ZFC.
6. Show that if two or more inaccessible cardinals exist, then the statement asserting the existence of two or more inaccessible cardinals is not a theorem of ZFC + "there exists at least one inaccessible cardinal".