Math 558 – Homework #1

January 29, 2007

1. Prove that the 1-place number-theoretic function

$$f(n) = F_n = \text{the } n\text{th Fibonacci number}$$

   is primitive recursive.

   Note: The Fibonacci numbers are given by the 2-step recursion $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Thus $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, $F_7 = 13$, $F_8 = 21$, $F_9 = 34$, $F_{10} = 55$, ...

2. Prove that the 2-place number-theoretic functions $\text{GCD}(x, y)$ and $\text{LCM}(x, y)$, the greatest common divisor and least common multiple of x and y, are primitive recursive.

3. Write a register machine program which computes the *exponential function*, i.e., the 2-place number-theoretic function $\exp(x, y) = x^y$. Note that $x^0 = 1$ for all x.

4. Let $f : \mathbb{N} \xrightarrow{1-1 \text{ onto}} \mathbb{N}$ be a permutation of $\mathbb{N}$, the set of natural numbers. Show that if f is register machine computable, then the inverse permutation f^{-1} is also register machine computable.

5. Prove that the 1-place number-theoretic function

$$f(n) = \text{the } n\text{th decimal digit of } \pi = 3.141592\cdots$$

   is register machine computable.