

Foundations of Mathematics

Math 558 — Final Exam

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May 2, 1997

Do two or three of the following:

1. State the Enumeration Theorem, and sketch its proof. In your sketch you may assume the following results: (i) every primitive recursive function is computable; (ii) the class of partial computable functions is closed under the least number operator.
2. State the Parametrization Theorem. Use it to prove:
 - (a) $\{e \mid \varphi_e \text{ is total}\}$ is non-recursive;
 - (b) the Recursion Theorem.
3. Explain the relationship between the arithmetical hierarchy Σ_k^0 , Π_k^0 , $k \in \omega$, and definability over the natural number system $(\mathbb{N}, +, \cdot, 0, 1, =)$. Use results about the arithmetical hierarchy to prove Tarski's theorem on undefinability of arithmetical truth.
4. Outline the proof of quantifier elimination for the real number system $(\mathbb{R}, +, -, \cdot, 0, 1, <, =)$.
5. State the Löwenheim–Skolem theorem, and sketch its proof. Explain how the Löwenheim–Skolem theorem can be used to obtain a countable transitive model of ZFC (under an appropriate hypothesis).
6. State what is meant by “the consistency of the Continuum Hypothesis.” Outline a proof of this relative consistency result.