

# Math 557 – Midterm Exam #1

October 8, 2003

1. Find a sentence in prenex normal form which is logically equivalent to  $(\forall x \exists y Rxy) \Rightarrow \neg \exists x Px$ .
2. Using the predicates  $Bx$  (“ $x$  is a barber in Podunk”) and  $Sxy$  (“ $x$  shaves  $y$ ”), translate the following argument into a sentence of the predicate calculus.

Any barber in Podunk shaves exactly those individuals who do not shave themselves. Therefore, there is no barber in Podunk.

Use an unsigned tableau to test this argument for logical validity.

3. Using the predicate  $Rxy$  (“ $x$  is an ancestor of  $y$ ”), translate the following argument into a sentence of the predicate calculus.

Every ancestor of an ancestor of an individual is an ancestor of the same individual. No individual is his own ancestor. Therefore, there is an individual who has no ancestor.

Is this argument valid? Justify your answer by means of an appropriate structure or tableau.

4. Let  $G$  be a group. The *order* of an element  $a \in G$  is the smallest positive integer  $n$  such that  $a^n = e$ . Here  $e$  denotes the identity element of  $G$ , and

$$a^n = \underbrace{a \cdot \dots \cdot a}_{n \text{ times}}.$$

Using only the predicates  $Pxyz$  (“ $x \cdot y = z$ ”) and  $Ixy$  (“ $x = y$ ”), write a sentence  $S$  of the predicate calculus such that, for any group  $G$ ,  $G$  satisfies  $S$  if and only if  $G$  has no elements of order 2 or 3.