

Homework # 2

August 28, 1998

1. Let X and Y be formulas.
 - (a) Show that X is a tautology iff $\sim X$ is not satisfiable.
 - (b) Show that X is satisfiable iff $\sim X$ is not a tautology.
 - (c) Show that X truth-functionally implies Y iff $X \supset Y$ is a tautology.
 - (d) Show that X is truth-functionally equivalent to Y iff $X \leftrightarrow Y$ is a tautology.
2. Prove the DeMorgan laws:
 - (a) $\sim (X \wedge Y) \simeq \sim X \vee \sim Y$.
 - (b) $\sim (X \vee Y) \simeq \sim X \wedge \sim Y$.
3. Show that if X is part of Y and if $X \simeq X_1$ and if Y_1 is obtained from Y by replacing X by X_1 , then $Y \simeq Y_1$. (Smullyan, exercise 1, page 13.)
4. Show that any formula is equivalent to a formula in disjunctive normal form. (Exercise 3, page 13.)
5. Show that \wedge is definable from \sim , \vee , etc. (Exercise 4, page 14.)
6. Show that all binary connectives are definable from the Sheffer stroke $|$, and from joint denial \downarrow . (Exercise 5, page 14.)