

Math 557 – Homework #8

November 14, 2003

1. Show that there exist groups G_1 and G_2 such that G_1 is a torsion group, G_1 is elementarily equivalent to G_2 , yet G_2 is not a torsion group. Deduce that there is no set of sentences S such that, for all groups G , G satisfies S if and only if G is a torsion group.
2. Show that there exist graphs G_1 and G_2 such that G_1 is connected, G_1 is elementarily equivalent to G_2 , yet G_2 is not connected. Deduce that there is no set of sentences S such that, for all graphs G , G satisfies S if and only if G is connected.
3. Show that the theory of fields of characteristic 0 is not finitely axiomatizable. I.e., there is no finite set of sentences S such that, for all fields F , F satisfies S if and only if F is of characteristic 0.
4. Let T be a 1-sorted theory. Assume that (a) T is consistent, (b) all models of T are infinite, and (c) there exists an infinite cardinal $\kappa \geq$ the cardinality of the language of T , such that T is κ -categorical.
Under these assumptions, show that T is complete.
(This is known as *Vaught's Test* for completeness.)
5. Which of the following theories are complete? Justify your answers.
 - (a) the theory of dense linear orderings with end points.
 - (b) the theory of fields of characteristic 0.
 - (c) the theory of infinite, torsion-free, Abelian groups.
 - (d) the theory of finite-dimensional vector spaces over a field of 5 elements.
 - (e) the theory of infinite-dimensional vector spaces over a field of 5 elements.