

Math 557 – Homework #7

October 22, 2003

1. Construct LH -derivations of the following logically valid sentences. To make this easier, you may supplement the rules of LH with

$$\frac{A_1 \cdots A_k}{B}$$

whenever B is a quasitautological consequence of A_1, \dots, A_k .

- (a) $(\forall x (Px \& Qx)) \Leftrightarrow ((\forall x Px) \& (\forall x Qx))$
- (b) $(\exists x \forall y Rxy) \Rightarrow (\forall x \exists y Ryx)$
- (c) $\neg \exists x \forall y (Eyx \Leftrightarrow \neg Eyy)$

2. The proof system LH' .

Consider the following proof system LH' , which is a “stripped down” version of LH . The objects of LH' are L - V -sentences containing only $\forall, \Rightarrow, \neg$ (i.e., not containing $\exists, \Leftrightarrow, \&, \vee$). The rules of LH' are:

- (a) quasitautologies
- (b) $(\forall x B) \Rightarrow B[x/a]$
- (c) $(\forall x (A \Rightarrow B)) \Rightarrow (A \Rightarrow \forall x B)$
- (d) $\frac{A \quad A \Rightarrow B}{B}$ (modus ponens)
- (e) $\frac{B[x/a]}{\forall x B}$ (generalization), where a does not occur in B .

Show that LH' is sound and complete.

3. The proof system $LH(S)$.

- (a) Let S be a set of L -sentences. Consider a proof system $LH(S)$ consisting of LH with additional rules of inference $\langle A \rangle, A \in S$. Show that an L - V -sentence B is derivable in $LH(S)$ if and only if B is a logical consequence of S .
- (b) Indicate the modifications needed when S is a set of L - V -sentences.