Math 557 – Homework #5

September 22, 2003

1. Use signed tableaux to show that the following are logically valid.

   (a) $(\forall x\,(A \Rightarrow B)) \Rightarrow ((\forall x\, A) \Rightarrow (\forall x\, B))$

   (b) $(\exists x\,(A \vee B)) \Leftrightarrow ((\exists x\, A) \vee (\exists x\, B))$

   (c) $(\exists x\, A) \Leftrightarrow (\neg \forall x\, \neg A)$

   (d) $(\forall x\,(A \vee C)) \Leftrightarrow ((\forall x\, A) \vee C)$, provided x is not free in C

2. Let L be the language consisting of one binary predicate, R. Recall that an L-*structure*, M, consists of a nonempty set $U_M = U$, the *domain* or *universe* of M, together with a binary relation $R_M = R \subseteq U \times U$. A set of L-sentences S is said to be *satisfiable* if there exists an L-structure M such that all of the sentences of S are true in M.

   Consider the following sentences:

   (a) $\forall x\, Rxx$

   (b) $\forall x\, \neg Rxx$

   (c) $\forall x\, \forall y\, (Rxy \Rightarrow Ryx)$

   (d) $\forall x\, \forall y\, (Rxy \Rightarrow \neg Ryx)$

   (e) $\forall x\, \forall y\, \forall z\, ((Rxy \,\&\, Ryz) \Rightarrow Rxz)$

   (f) $\forall x\, \exists y\, Rxy$

   Which of subsets of this set of sentences are satisfiable? Verify your claims by exhibiting appropriate structures. Use the simplest possible structures.