

Math 557 – Homework #2

September 10, 2003

1. Let C be the formula $(p \& \neg q) \Rightarrow \neg(p \vee r)$.
 - (a) Restore all the omitted parentheses to C . (See Remark 1.1.5.)
 - (b) Exhibit a formation sequence for C .
 - (c) List the immediate subformulas of C , their immediate subformulas, etc., i.e., all subformulas of C .
 - (d) Calculate the degrees of C and its subformulas.
 - (e) Display the formation tree of C .
 - (f) Write C according to various notation systems:
 - i. The rules 1–3 of Definition 1.1.3:
 1. Each atom is a formula.
 2. If A is a formula then $(\neg A)$ is a formula.
 3. If A and B are formulas and b is a binary connective, then (AbB) is a formula.
 - ii. The following alternative set of rules:
 1. Each atom is a formula.
 2. If A is a formula then $\neg(A)$ is a formula.
 3. If A and B are formulas and b is a binary connective, then $(A)b(B)$ is a formula.
 - iii. Polish notation.
 - iv. Reverse Polish notation.
2. Use truth tables to show that $((A \Rightarrow B) \Rightarrow A) \Rightarrow A$ is logically valid.
3. Use truth tables to show that $(A \& B) \Rightarrow C$ is logically equivalent to $A \Rightarrow (B \Rightarrow C)$.
4. Prove the following. (See Remarks 1.2.13 and 1.3.2.)
 - (a) B is logically valid if and only if $\neg B$ is not satisfiable.
 - (b) B is satisfiable if and only if $\neg B$ is not logically valid.
 - (c) B is a logical consequence of A_1, \dots, A_n if and only if $(A_1 \& \dots \& A_n) \Rightarrow B$ is logically valid.

(d) A is logically equivalent to B if and only if $A \Leftrightarrow B$ is logically valid.

5. Formulate the following argument as a propositional formula.

If it has snowed, it will be poor driving. If it is poor driving,
I will be late unless I start early. Indeed, it has snowed.
Therefore, I must start early to avoid being late.

6. Use the tableau method to demonstrate that this formula is logically valid.

7. Brown, Jones, and Smith are suspected of a crime. They testify as follows:

Brown: Jones is guilty and Smith is innocent.

Jones: If Brown is guilty then so is Smith.

Smith: I'm innocent, but at least one of the others is guilty.

Let b , j , and s be the statements "Brown is innocent," "Jones is innocent," "Smith is innocent". Express the testimony of each suspect as a propositional formula. Write a truth table for the three testimonies.

8. Use the above truth table to answer the following questions:

(a) Are the three testimonies consistent?

(b) The testimony of one of the suspects follows from that of another. Which from which?

(c) Assuming everybody is innocent, who committed perjury?

(d) Assuming all testimony is true, who is innocent and who is guilty?

(e) Assuming that the innocent told the truth and the guilty told lies, who is innocent and who is guilty?

9. Use a signed tableau to show that $(A \Rightarrow B) \Rightarrow (A \Rightarrow C)$ is a logical consequence of $A \Rightarrow (B \Rightarrow C)$.

10. Use a signed tableau to show that $A \Rightarrow B$ is logically equivalent to $(\neg B) \Rightarrow (\neg A)$.

11. Use an unsigned tableau to show that $A \Rightarrow (B \Rightarrow C)$ is logically equivalent to $(A \& B) \Rightarrow C$.

12. Use an unsigned tableau to test $(p \vee q) \Rightarrow (p \& q)$ for logical validity. If this formula is not logically valid, use the tableau to find all assignments which falsify it.
13. Redo the previous problem using a signed tableau.