Math 557 – Homework #6

September 22, 2000

1. (10 points)

   Let L be the language consisting of one binary predicate, R. Recall that an L-*structure*, M, consists of a nonempty set $U_M = U$, the *domain* or *universe* of M, together with a binary relation $R_M = R \subseteq U \times U$. A set of L-sentences S is said to be *satisfiable* if there exists an L-structure M such that all of the sentences of S are true in M.

   Consider the following sentences:

   (a) $\forall x\, Rxx$

   (b) $\forall x\, \neg\, Rxx$

   (c) $\forall x\, \forall y\, (Rxy \Rightarrow Ryx)$

   (d) $\forall x\, \forall y\, (Rxy \Rightarrow \neg\, Ryx)$

   (e) $\forall x\, \forall y\, \forall z\, ((Rxy \,\&\, Ryz) \Rightarrow Rxz)$

   (f) $\forall x\, \exists y\, Rxy$

   Which of subsets of this set of sentences are satisfiable? Verify your claims by exhibiting appropriate structures. Use the simplest possible structures.

2. (10 points)

   Use signed tableaux to show that the following are logically valid.

   (a) $(\forall x\, (A \Rightarrow B)) \Rightarrow ((\forall x\, A) \Rightarrow (\forall x\, B))$

   (b) $(\exists x\, (A \vee B)) \Leftrightarrow ((\exists x\, A) \vee (\exists x\, B))$

   (c) $(\exists x\, A) \Leftrightarrow (\neg\, \forall x\, \neg\, A)$

   (d) $(\forall x\, (A \vee C)) \Leftrightarrow ((\forall x\, A) \vee C)$, provided x is not free in C